Plastic Hinge Length for Lightly Reinforced C-shaped Concrete Walls

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Abstract

This research investigates the equivalent plastic hinge length of reinforced concrete C-shaped walls with reinforcement detailing typically found in low-to-moderate seismic regions. Reinforced concrete walls in these regions commonly have low amounts of longitudinal reinforcement and unconfined boundary regions, which have been shown to perform poorly in recent earthquake events. A series of state-of-the-art finite element analyses are undertaken to find the longitudinal strain distributions of low-rise, mid-rise and high-rise C-shaped walls. The results of the equivalent plastic hinge lengths from the numerical investigation are shown to compare poorly to the predictions from some of the equations that currently exist in the literature. Subsequently, expressions are derived for the equivalent plastic hinge length for these types of walls and for the different modes of bending. The expressions derived from this research intend to improve the displacement capacity for these types of walls when using plastic hinge analyses.

1. Introduction

Reinforced concrete (RC) walls are commonly used to provide the primary resistance for buildings when subjected to lateral loading caused by wind or earthquakes. In regions of low-to-moderate seismicity, such as Australia, the majority of the RC walls are lightly reinforced (Hoult et al., 2017a; Wibowo et al., 2013; Wilson et al., 2015). Past and recent research conducted on the seismic performance of lightly reinforced concrete walls has typically focused on rectangular sections (Altheeb, 2016; Cardenas &
Magura, 1972; Lu et al., 2016; Oesterle et al., 1976; Puranam & Pujol, 2017). However, RC walls (or cores) are often non-rectangular due to structural and architectural requirements (Belletti et al., 2013; Smyrou et al., 2013). For example, a C-shaped wall is a popular construction choice as it can enclose elevators or stairs (Beyer et al., 2008a). The behaviour of C-shaped walls can also differ considerably in comparison to rectangular walls when subjected to lateral loading. This is discussed in Smyrou et al. (2013) for a T-shaped wall, where the configuration of the wall and direction of loading can significantly affect the stiffness, strength and ductility. These observations also relate to C-shaped wall sections; for bending about the minor axis, which causes the web of the C-shaped wall to go into compression (WiC), the reinforcement in the flanges is likely to yield and this results in substantial plastic deformations. This is due to the relatively small depth required from the neutral axis to the extreme fibre edge in compression, which means large tension strains govern for this direction of loading. However, when the web of the C-shaped wall is in tension (WiT), a non-ductile and brittle failure can occur due to the large (governing) compression strains (and stresses) needed to balance the tensile forces developed by the web’s longitudinal reinforcement. These loading scenarios and idealised strain distributions are illustrated in Figure 1, which are similar to the observations made in Paulay and Priestley (1992).
Figure 1 (a) compression governing (WiT) and (b) tension governing (WiC) C-shaped walls

Poor performance associated with the seismic performance of lightly reinforced walls has been observed in recent earthquake events (CERC, 2012; Wood et al., 1991). Most notably, a single crack has been found to form in the plastic hinge zone of lightly reinforced walls leading to large strain concentrations in the reinforcement at this crack and potentially, with enough cycles, fracturing of the reinforcement. For instance, it was likely that the RC core walls of the Pyne Gould Corporation Building, shown in Figure 2, had an insufficient amount of longitudinal reinforcement to transmit the required tension to initiate secondary cracking in the surrounding concrete (CERC, 2012). Thus, in comparison to the expected distribution of cracks (and corresponding distribution of strains) up a significant portion of the wall height, as is usually assumed when determining the plastic hinge length, the yielding of reinforcement was ‘confined to a short length resulting in a single wide crack in the potential plastic region at level 1’ (CERC, 2012). The Pyne Gould building collapsed in a non-ductile, brittle and catastrophic fashion during the February 22nd 2011 Christchurch earthquake. The mean of the measured in-situ concrete
strength \( f_{cm} \) from six concrete cores extracted from part of the RC core remnants of the Pyne Gould building from Beca (2011) was calculated to be 26 MPa. The longitudinal and horizontal reinforcement in the 203 mm thick walls of the Pyne Gould Building consisted of a single grid of 16 mm diameter bars at 381 mm centres (Beca, 2011), corresponding to longitudinal and transverse reinforcement ratios \( \rho_{wv} \) and \( \rho_{vh} \) respectively of approximately 0.27%. The mean yield \( f_y \) and ultimate strength \( f_u \) of the longitudinal deformed 16 mm reinforcing steel bars measured from some remnants of the RC walls that were reported in Beca (2011) were calculated to be 307 MPa and 432 MPa respectively. An expression (Equation 1) is given in Hoult et al. (2017a) to determine if a RC wall has a sufficient amount of longitudinal reinforcement to allow secondary cracking. Using Equation 1 and the values above, it is estimated that the minimum longitudinal reinforcement ratio \( \rho_{wv,min} \) required of the Pyne Gould building core walls to allow secondary cracking was approximately 0.60%. This means that the RC core walls of the Pyne Gould building had an insufficient amount of longitudinal steel, which corroborates with the failure mode of the wall as reported in CERC (2012). It should be noted that the flexural tensile strength of the concrete \( f'_{ct,\beta} \) was calculated using the expressions recommended in fib (2012) (Equations 2 through to 4), which uses the direct tensile strength \( f_d \). Furthermore, the \( f_{cm} \) value was used in Equation 2, substituting the value of the characteristic cylinder strength at 28-days \( f'_c \), as the focus of this investigation was on assessment.

\[
\rho_{wv,min} = \frac{(t_w - n_{br}d_{br})f_{ct,ft}}{f_ut_w}
\]

where \( t_w \) is the thickness of the wall, \( n_{br} \) is the number of grids of the transverse reinforcement (corresponding to 1 in the case of the Pyne Gould core walls) and \( d_{br} \) is the diameter of the transverse reinforcement.

\[
f_{ct} = 0.3(f'_c)^{2/3}
\]
\[
\alpha_f = \frac{0.06h^{0.7}}{1 + 0.06h^{0.7}}
\]

where \( h \) (in mm) is the depth (e.g. \( L_w \)) of the section

\[f_{ct,fl} = \frac{f_{ct}}{\alpha_f}\]

Figure 2 Plan view of the Pyne Gould Corporation Building for the levels above the ground floor (CERC, 2012)

In Australia and other low-to-moderate seismic regions, it is believed that RC walls with \( \rho_{aw} \) less than 2.00% ‘represent the great majority of [the] building stock’ (Wibowo et al., 2013). In fact, Adebar and Lorzadeh (2012) note that it is rare for RC walls to have a \( \rho_{aw} \) greater than 1.00%. The Concrete Structures code in Australia AS 3600:2009 (Standards Australia, 2009) requires a RC structural wall to have the minimum longitudinal reinforcement ratio of just 0.15%. However, the AS 3600:2009 also requires the ultimate design strength in bending of RC beams to be larger than 1.2 of the cracking moment \( (M_{cr}) \), which is a common code provision (Priestley et al., 1996). This design check also applies
to RC walls that are designed in accordance with AS 3600:2009 if the wall is subjected to ‘in-plane horizontal forces’ in conjunction with ‘axial forces’, such that part of the wall is expected to be in tension. Therefore, it is likely that the minimum $\rho_{\text{av}}$ of 0.15% would increase if this design check was met.

However, recent research has shown that the ultimate moment capacity ($M_u$) of the wall is required to be much larger than $1.2M_{cr}$ to allow secondary cracking, with values typically ranging from $2.0M_{cr}$ to $2.5M_{cr}$ (Henry, 2013; Hoult et al., 2017a). Therefore, it is likely that many of the structural walls embedded in the RC building stock of Australia have been designed with an insufficient amount of longitudinal reinforcement to allow secondary cracking.

While it was estimated that the west wall of the Pyne Gould core yielded in vertical tension between levels one and two, Beca (2011) estimate that the east wall failed disastrously in vertical compression. These core walls had no boundary elements or confinement, as this was not a requirement of the design codes at the time of construction. This is also highlighted in CERC (2012), where ‘the wall lacked the confining reinforcing needed to provide the ductility required to withstand the extreme actions that resulted from the February 2011 aftershock’. Poor performance of non-rectangular RC walls was also observed in major populated centres in Chile following an earthquake in 2010. Wallace et al. (2012) particularly emphasised the inadequate behaviour exhibited by poorly detailed and/or compression-controlled RC walls that had insufficient confinement to withstand a ‘stable compression zone and ensure spread of plasticity by confining core concrete and supressing rebar buckling’. This is of major concern for non-rectangular RC walls in low-to-moderate seismic regions, which can (i) be prone to a compression controlled performance (Figure 1), and (ii) lack confinement in the boundary regions of the wall, which is typically not a requirement in the current building codes and Standards (e.g. AS 3600:2009).

Although C-shaped walls are prevalent throughout the RC building stock of regions of low-to-moderate seismicity, there have been very few studies and experimental testing conducted on non-rectangular RC walls (Beyer et al., 2008a; Constantin & Beyer, 2014). The authors acknowledge some testing has been conducted on some RC C-shaped wall sections (Beyer et al., 2008b; Constantin & Beyer, 2016; Lowes et
al., 2013; Menegon et al., 2017; Reynouard & Fardis, 2001; Sittipunt & Wood, 1993). However, the large majority of these C-shaped wall sections have high longitudinal reinforcement ratios, lumping of reinforcement and confined boundary regions. There is a paucity of experimental research that focuses on lightly reinforced and unconfined C-shaped walls that represent typical sections found in low-to-moderate seismic regions, such as Australia. Beyer (2007) noted that it would be ‘interesting to investigate the behaviour of U-shaped [or C-shaped] walls in which such boundary elements are either missing or poorly detailed’, as the author believed that ‘the boundary elements at the corners were essential for the ductile behaviour of the U-shaped walls’. Moreover, the test results from Constantin (2016) further emphasised the importance of ‘proper confinement of the flange ends to ensure the wall displacement ductility’.

Therefore, an investigation on the seismic performance of C-shaped walls with detailing commonly found in low-to-moderate seismic regions is warranted. For the sake of brevity, this paper focuses solely on the longitudinal strain distribution and plastic hinge length of lightly reinforced and unconfined C-shaped concrete walls. It should be noted that the authors intend to investigate and address the important issue of estimating the displacement capacity, using plastic hinge analyses, of lightly reinforced rectangular and C-shaped concrete walls, which will be reported in a separate research paper. It is likely that many of the current expressions available in the literature for plastic hinge analysis will overestimate the displacement capacity of these types of walls, as these expressions have been primarily derived from experimental and numerical studies on walls designed for high seismic regions (Hoult et al., 2016). The results from these studies will provide vital information for the vulnerability studies of RC buildings that are being carried out by the authors within the earthquake mitigation component of the Australian CRC (Cooperative Research Centre) for Bushfires and Natural Hazards.

2. Equivalent Plastic Hinge Length

A Plastic Hinge Analysis (PHA) is one of the most widely used and simplest methods for calculating the force-displacement capacities of RC members (Almeida et al., 2016). To calculate the plastic deformation ($\Delta_p$) component of the wall due to flexure, the strains and corresponding curvatures need to
be determined, and these will vary considerably up to the height of a cantilevered wall. However, it is
common practice to simply assume that the inelastic curvature is uniform for a height above the base that
is equivalent to the plastic hinge length ($L_p$) (Fenwick & Dhakal, 2007). This is a reasonable
approximation and is utilised, for example, in the equations proposed by Priestley et al. (2007) for the
displacement profile of a cantilevered wall.

Many empirical equations are available for calculation of the equivalent plastic hinge length ($L_p$). While
there are many empirical equations that are available for calculation of the equivalent plastic hinge length
($L_p$) (Bohl & Adebar, 2011; Kazaz, 2013; Moehle, 1992; Paulay, 1986; Priestley & Park, 1984; Priestley
et al., 2007; Priestley & Park, 1987; Thomsen & Wallace, 2004; Wallace & Moehle, 1992), the great
majority of these have been derived from experimental or numerical investigations on beams, columns or
walls that are highly confined, contain large amounts of longitudinal reinforcement and use reinforcing
steel with mechanical properties that would not be used in regions of low-to-moderate seismicity
(Ilannewald, 2013; Hoult et al., 2017a). For example, Equation 5 is a widely used expression from
Priestley et al. (2007). Moreover, Constantin (2016) derived a plastic hinge length expression (Equation
7) specifically for RC C-shaped walls detailed to regions of high seismicity that was calibrated using
experimental and numerical analyses. There has also been some recent research focusing on the $L_p$ of RC
walls with inferior detailing. For example, the expression from Hoult et al. (2017a) (Equation 8) was
derived specifically for lightly reinforced and unconfined rectangular walls. These equations will later be
scrutinised as to their applicability for lightly reinforced and unconfined C-shaped walls.

$$L_p = kH_e + 0.1L_w + L_{sp}$$

where $k$ is a constant reflecting the distribution of plasticity dependent on the ratio of the ultimate strength
to yield strength of the reinforcing steel ($k = 0.2(f_u/f_y - 1) \leq 0.08$), $L_w$ is the wall length, and $L_{sp}$ is the strain
penetration length defined in Priestley et al. (2007) with Equation 6.
\[ L_{sp} = 0.022 f_{ye} d_{bl} \]

\[ L_p = \left[ 0.05 H_e + 0.05 L_w \left( \frac{\tau}{0.17 \sqrt{f'_c}} \right) \right] (1 + 4 ALR) \]

where \( \tau \) is the average shear stress parameter, which can be calculated from a sectional analysis (moment-curvature analysis) or can be estimated by using a simplified approach, which involves dividing the base shear \( V_b \) of the wall by the gross cross-sectional area of the wall \( (A_g) \) (Krolicki et al., 2011). It should be noted that in Equation 7, Constantin (2016) has taken \( L_w \) as the length of the wall parallel to the direction of loading; for example, the \( A_g \) corresponding to the length of the web \( (L_{web}) \) is used for bending about the major axis (and neglecting the cross-section from the flanges). Moreover, the average shear stress for bending about the minor axes was calculated using the \( A_g \) from both of the flanges (and neglecting the web of the wall), where the ‘shear force in the direction of the flanges [were] equally distributed between the flanges’ (Constantin, 2016).

\[ L_p = (0.10 L_w + 0.075 H_e)(1 - 6.0 \left( \frac{P}{A_g f_{cml}} \right)) \leq 0.5 L_w \]

The \( L_p \) can be determined from the results of numerical analyses using the Equivalent Plastic Hinge Length (EQPL) method. This method has been used in past studies (Fenwick & Dhakal, 2007; Henry, 2013; Hoult et al., 2017a; Mortezaei & Ronagh, 2012) and has also been used to derive the plastic hinge length and curvature limits set in NZS 3101:2006 (Standards Association NZ, 2006). The EQPL method uses the curvature distribution up the height of the cantilever wall and the rotation at the base of the wall; as is well known from the first moment area theorem, the rotation between two points up the height of the wall is equal to the area under the curvature distribution. The curvature distribution, and similarly for the rotations, can be idealised into elastic and plastic regions. The primary purpose of this study is to
determine the extent of plasticity occurring at the wall base. Hence, the EQPL method will be used to
estimate the $L_p$ from the curvature distributions of the C-shaped walls analysed in VecTor3.

3. VecTor3

VecTor3 (ElMohandes & Vecchio, 2013) is a state-of-the-art nonlinear finite element modelling program
used for RC 3-dimensional solids that uses the Disturbed Stress Field Model (DSFM) (Vecchio et al.,
2000). Information about the program and some recent improvements of the predictions for the behaviour
of RC elements can be found in (ElMohandes, 2013).

The next section introduces the materials models that be used in VecTor3. Following this, a ductile C-
shaped wall specimen from Beyer et al. (2008b) and (importantly) a limited-ductile C-shaped wall
specimen from Menegon et al. (2017) are used to validate the material models. A sensitivity study is also
conducted to observe any discrepancies in using different element sizes affecting the estimated force-
displacement relationship and the equivalent plastic hinge length. Once VecTor3 has provided good
estimates to these C-shaped wall specimens, the program will be used to analyse a large number of lightly
reinforced and unconfined C-shaped walls with varying parameters to find the longitudinal strain
distribution (up the wall height) such that the equivalent plastic hinge length can be calculated.

3.1 Material Models

A range of material models have been incorporated in VecTor3 to represent modelling of the concrete and
steel materials. These models were used for the finite element modelling analyses conducted in Hoult et
al. (2017a) for lightly reinforced and unconfined rectangular walls. Subsequently, they are used here for
modelling C-shaped walls with similar detailing. These models are summarised in Table 1, where the
reader is directed to Hoult et al. (2017a) and ElMohandes and Vecchio (2013) for more information on
the selected models.

<table>
<thead>
<tr>
<th>Constitutive Behavior</th>
<th>Model</th>
</tr>
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</table>

Table 1: Constitutive models to be used in the VecTor3 analyses
3.2 Comparisons to Experimental Analyses

In this section, VecTor3 will be used to model two RC C-shaped walls in an attempt to validate the chosen materials models (Table 1) for the program such that the proposed lightly reinforced and unconfined C-shaped walls (Section 3.3) can be analysed in VecTor3 to a high degree of accuracy. While only two C-shaped walls are used to validate the material models here, it should be emphasised that the results from modelling some lightly reinforced rectangular RC wall specimens in VecTor3 using these same material models have shown good correlations with the experimental observations (Hoult, 2017).

3.2.1 TUA from Beyer et al. (2008b)

Beyer et al. (2008b) investigated the seismic performance and inelastic behaviour of two U-shaped (or “C-shaped”) RC walls. One of the two wall specimens, denoted “TUA”, is shown in Figure 3 and was built at half-scale with detailing for high ductility. The concrete cylinder strength of TUA was much higher than the intended strength that was aimed for ($f'_c = 45$ MPa), resulting in a mean insitu concrete strength ($f_{cm}$) value of 77.9 MPa (Beyer, 2007). A ductile steel was used for the reinforcement, with a $f_u / f_y$ ratio of 1.22 (or 595 MPa / 488 MPa) and $\varepsilon_u$ of 13% being used for the longitudinal D12 bars. Using

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1 Normal Strength Concrete (NSC) < 45MPa
2 High Strength Concrete (HSC) ≥ 45MPa
the expressions given in Section 1, the $\rho_{ov,min}$ for TUA is approximately 0.87%. The $\rho_{ov}$ for the boundary regions of TUA (Figure 3) is approximately 2.00%, which means that secondary cracking is expected to occur for this wall. The effective height ($H_e$) of the specimen differed depending on the applied loading direction, with a $H_e$ of 3.35 m and 2.95 m for the walls bending about the major and minor axes respectively. A quasi-static loading protocol was used to assess the walls’ behaviour different directions of loading. The results from the experiment showed that the design approach adopted by Beyer et al. (2008b) resulted in a ductility ($\mu$) of approximately 8 for specimen TUA. This was not surprising given the quality of the detailing used, with a relatively high longitudinal reinforcement ratio, lumped reinforcement in the “boundary” regions of the wall and high levels of confinement. This type of detailing is uncommon in Australia and other regions of low-to-moderate seismicity. More information on the instrumentation used, test setup and cyclic loading history can be found in Beyer et al. (2008b) and Beyer (2007).

![Figure 3 Cross-section of wall specimen TUA from Beyer et al. (2008b)](image-url)
VecTor3 (ElMohandes & Vecchio, 2013) was used to model wall specimen TUA from Beyer et al. (2008b). Two different mesh setups were used to evaluate if there were likely to be any discrepancies in the performance predicted by VecTor3 due to element size. One of the models used 2 elements across the thickness of the wall (Model 2E) and the other used 3 elements (Model 3E). The same recommendation from Palermo and Vecchio (2007) that was used to limit the element sizes in Vector 2 to a 3:2 aspect ratio was used in VecTor3, but extrapolated here for elements in 3-dimensions. Therefore, the element size in the z-direction was also limited to an aspect ratio of 3:2 using the smaller of the element size in the x or y direction. This rule was adhered to in both of the models. The two models have the element sizes, total number of nodes \(n_i\) and number of elements \(e_i\) given in Table 2; \(s_x, s_y\) and \(s_z\) correspond to the element dimensions in the x, y and z-directions respectively. A refined mesh size was used for the lower region of the wall in a similar manner to the approach adopted in Hoult et al. (2017a) for rectangular walls that were modelled with VecTor2; the hexahedral elements above this region have an incrementally increasing vertical (z-direction) mesh size to reduce the number of nodes and elements required. This decreased the computation time while improving the accuracy of cracking and strain distributions (vertically and horizontally) at the base of the wall. This approach has also been used successfully by other researchers (Bohl & Adebar, 2011; Hoult et al., 2017a; Kwan, 1996). The model with 2 elements across the thickness (Model2E) is illustrated in Figure 4. Four different concrete materials that consisted of different amounts of smeared vertical and horizontal reinforcement were used to represent the web, flanges, intersection of web and flanges and the boundary ends of the flanges. The corresponding longitudinal (in the z-direction) and transverse (in the x and y-directions) reinforcement ratios used in these different regions are shown in Figure 4. The mechanical properties of the 6 mm diameter \(d_{n}\) longitudinal bars that are used in the unconfined web or flange regions of the wall are given in Table 3. Moreover, the mechanical properties of the 12 mm \(d_{n}\) longitudinal bars that are used in the confined boundary regions of the wall are also given in Table 3. It should be noted that 0.6 of the reported \(e_{uw}\) was adopted for the ultimate strain of steel, as recommended by Priestley et al. (2007) to account for the possibility of low cycle fatigue.
Table 2 Nodes and elements used for the two different models for TUA (Beyer et al., 2008b)

<table>
<thead>
<tr>
<th></th>
<th>2 elements</th>
<th>3 elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_x$</td>
<td>75 mm</td>
<td>50 mm</td>
</tr>
<tr>
<td>$s_y$</td>
<td>75 mm</td>
<td>50 mm</td>
</tr>
<tr>
<td>$s_{z1}$</td>
<td>75 mm</td>
<td>75 mm</td>
</tr>
<tr>
<td>$s_{z2}$</td>
<td>100 mm</td>
<td>100 mm</td>
</tr>
<tr>
<td>$s_{z3}$</td>
<td>150 mm</td>
<td>150 mm</td>
</tr>
<tr>
<td>$n_f$</td>
<td>3780</td>
<td>7560</td>
</tr>
<tr>
<td>$e_f$</td>
<td>2378</td>
<td>5394</td>
</tr>
</tbody>
</table>

Figure 4 C-shaped wall specimen TUA in VecTor3 (Model 2E)

Table 3 Mechanical properties of the reinforcing steel for TUA

<table>
<thead>
<tr>
<th>$d_{bl}$ (mm)</th>
<th>$f_y$ (MPa)</th>
<th>$f_u$ (MPa)</th>
<th>$\varepsilon_{fu}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>518</td>
<td>681</td>
<td>50.4</td>
</tr>
<tr>
<td>12</td>
<td>488</td>
<td>595</td>
<td>75.6</td>
</tr>
</tbody>
</table>

The nodes at the base of wall were assigned fixed conditions, while the nodes around the outer faces of the wall had constraints such that the wall would not move out of plane (e.g. the wall was fixed in the x-direction for lateral movement in the y-direction, and vice versa). A 1 mm displacement was applied to
all nodes at the top of wall for the lateral loading, which was then controlled incrementally with
monotonic loading. While this loading condition differs from the conditions used experimentally, which
involved applying actuators at a “collar” of increased thickness at the top of the wall, it is expected that
this simplified modelling condition in VecTor3 will not lead to any significant difference in results.
Furthermore, this same type of loading scheme was successfully employed in Hoult et al. (2017a) for
rectangular walls in VecTor2. The axial load was held constant during the analyses and was distributed
evenly to all of the nodes at the top of the wall section.

The force-displacement results from VecTor3 for all modes of bending for Model2E and Model3E are
given in Figure 5 and Figure 6 respectively. Reverse cyclic loading was also conducted in VecTor3 for
Model2E and for bending about the major axis, shown in Figure 5(a). The experimental results from
Beyer et al. (2008b) are superimposed in Figure 5 and Figure 6. In general, the force-displacement
relationship predicted by VecTor3 compares well with the experimental observations for both Model 2E
and 3E for all modes of bending.

![Image](figure5.png)

**Figure 5** Force-displacement results from VecTor3 for TUA Model2E and for bending about the (a) major
and (b) minor axes.
The experimental $L_p$ values were calculated and reported in Beyer (2007) for specimen TUA as a function of the wall ductility ($\mu$) and for the two directions of loading. Using the VecTor3 results for both Model2E and Model3E and for monotonic loading, concrete and steel strains in the extreme fibre regions up the wall height were extracted at the ultimate displacement. Using the EPHIL method (discussed Section 2.2), the $L_p$ was calculated for all 3 modes of bending. The resulting $L_p$ values are given in Table 4, and these are compared to the experimentally determined $L_p$ value at a $\mu$ of 4. It is important to note that after the wall exceeds $\mu$ values of approximately 4, the $L_p$ determined experimentally was relatively constant. There is little variance between the calculated $L_p$ values from VecTor3 for Model2E and Model3E and for the three different directions of loading. Moreover, the calculated $L_p$ from VecTor3 generally compares well to the experimentally determined $L_p$, particularly for bending about the major axis. The prediction from the VecTor3 results of a higher $L_p$ in comparison to the experimental results for bending about the minor axis with WiC is potentially a result of a slight overestimation of the tension strains in the extreme tension fibre region. However, this predicted $L_p$ value does not greatly overestimate what was determined experimentally.

**Table 4 Equivalent plastic hinge length values determined numerically and experimentally for TUA**

<table>
<thead>
<tr>
<th>Experimental</th>
<th>Numerical (Model2E)</th>
<th>Numerical (Model3E)</th>
</tr>
</thead>
</table>

URL: http://mc.manuscriptcentral.com/ueqe Email: gencturk@usc.edu
The experimentally determined curvature distributions (up to the wall height) at yield for wall specimen TUA were reported in Beyer (2007) and are given in Figure 7. The curvatures were obtained from strain measurements determined experimentally by Linear Variable Differential Transformers (LVDTS) at the edges of the wall. Similarly, average curvatures at yield were calculated from the VecTor3 results (Model2E) using the strain values at the edges of the wall (in the respective extreme fibre regions of the wall) and are superimposed in Figure 7. The curvature distributions determined from VecTor3 provide a reasonable comparison to that observed experimentally.

![Curvature distribution comparisons](image)

**Figure 7** Curvature distribution comparisons for wall TUA and for bending about the (a) major axis (b) minor axis

VecTor3 has given results that compare well to the experimental results from Beyer et al. (2008b) and Beyer (2007) using material models suitable for the well-detailed C-shaped walls that were tested, and
having 2 elements across the thickness of the flange. This has indicated that VecTor3 is capable of predicting the seismic performance of C-shaped walls. However, the C-shaped walls that will be investigated in VecTor3 in this study are to have similar detailing to that used for the rectangular walls in Hoult et al. (2017a); that is, the walls will have detailing typical of that found in Australia and other low-to-moderate seismic regions. The C-shaped specimens tested by Beyer et al. (2008b) represent walls with detailing that is common in high seismic regions. Therefore, it is also important to show that VecTor3, using the same material models used in this section for comparison with the experimental results of a well detailed wall, is capable of predicting the seismic performance of limited-ductile C-shaped walls.

3.2.2 S02 from Menegon et al. (2017)

Menegon et al. (2017) reported on an experimental program which was conducted to examine the in-plane seismic behaviour of RC walls with limited ductile detailing. Importantly, Menegon et al. (2017) tested a C-shaped core wall (box-shaped building core with door opening) that was ‘designed to best represent current building practices in Australia’. Wall specimen S02, with cross-section and elevated view shown in Figure 8, was designed to represent the ground floor (one storey) of a four storey core with limited ductility that was approximately 70% full scale. Thus, a moment and lateral force was subjected to the specimen during seismic testing to simulate a taller wall, which was also practice by Lowes et al. (2013) for several C-shaped wall tests. Menegon et al. (2017) estimated the shear-span-ratio, also equivalent to the aspect ratio \( A_r = H_c/L_w \), to be 6.5. Thus, while the test specimen height was only 2600 mm, using the web length of 1200 mm for \( L_w \), the effective height \( (H_c) \) of the wall was approximately 7800 mm.
The measured compressive strength was reportedly 31.6 MPa and the tensile strength was calculated to be 3.3 MPa using Equation 2 to 4 from fib (2012). Wall specimen S02 had evenly distributed longitudinal and transverse reinforcement using 12 mm and 10 mm diameter bars respectively. This corresponded to longitudinal and transverse reinforcement ratios of 1.4% and 0.5% respectively. The measured values for the mechanical properties of the D500N reinforcement used is given in Table 5. The ultimate strain of the steel was taken as $0.6\varepsilon_{fu}$ as discussed previously in Section 3.2.1. The wall specimen was tested under unidirectional quasi-static cyclic conditions. Only testing about its strong axis (bending about the major axis) was conducted, and so the comparison from the numerical analyses is limited to a single direction of motion. A constant axial load of 1200 kN was applied and maintained for the duration of the test, which corresponded to an $ALR$ of 7.7%.

| Table 5 Mechanical properties of the D500N bars used in specimen S02 |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $d_{sl}$ (mm) | $f_y$ (MPa) | $f_u$ (MPa) | $\varepsilon_{fu}$ (mm/m) | $0.6\varepsilon_{fu}$ (mm/m) |
|----------------|-----------|-----------|-----------------|-----------------|-----------------|
| 10             | 544.9     | 680.1     | 121             | 73              |
| 12             | 544.2     | 698.1     | 110             | 66              |

The VecTor3 model of wall specimen S02 is shown in Figure 9. Based on the results from modelling wall specimen TUA from the previous section (Section 3.2.1), it was decided to use 2 elements across the
thickness of the wall. Besides, given the size of this model, providing 3 elements across the thickness of
the wall would require a large and onerous total number of elements that would be bordering on the
limitations of the VecTor3 program. Furthermore, it was found to be too troublesome to mesh the wall
section above the door [as shown in Figure 8(a)], thus the VecTor3 model of specimen S02 was treated as
a C-shaped wall section throughout the height of the wall. The size of the elements ($s_x$, $s_y$ and $s_z$) for the
lower portion of the wall were 65 x 65 x 90 (in mm). It should be noted that the elements in the web were
slightly decreased in the direction parallel to the web ($s_x$) to 58.75 mm in order to obtain the actual length
of the web. Furthermore, the total wall length of the flange was slightly increased in the VecTor3 model
to 1235 mm (in comparison to the specimen that was 1200 mm) in order to use the warranted mesh
layout. The 3:2 aspect ratio for plane stress elements as recommended in Wong and Vecchio (2002) and
Palermo and Vecchio (2007) was adhered to for all of the elements in the lower portion of the wall. The
size of the elements outside of the predicted yielding zone (above a conservative height of 4500 mm)
were increased in the direction parallel to the wall height ($s_y$) to 191 mm to decrease the number of
required elements. The foundation was not modelled and all nodes at the base of the wall were assigned
fixed conditions, which was also practiced by Constantin (2016) in modelling C-shaped walls. The
VecTor3 model utilised smeared reinforcement in the concrete materials for the horizontal and vertical
bars in the wall. An axial load of 1200 kN was distributed evenly across all nodes at the top of the wall
and held constant throughout the analysis. The nodes at a height of 7747 mm, corresponding to the
approximate $H_c$, were subjected to a lateral displacement for both monotonic and reverse cyclic loading
scenarios.
Figure 9 VecTor3 model of wall specimen S02 (a) elevated 3D model and (b) cross-section

The force-displacement results from the VecTor3 simulations for both monotonic and reverse cyclic are presented in Figure 10. It should be noted that the displacement and drifts presented in Figure 10 from VecTor3 were taken at a height from the base of approximately 2600 mm to coincide with the height of the actual test specimen. Furthermore, the analyses stopped once a drift of 2.3% at this height of 2600 mm was achieved, which corresponded to the approximated drift prior to axial load failure of the specimen. Superimposed on Figure 10 is the experimental results from Menegon et al. (2017). The results from VecTor3 generally correlate well to the force-displacement hysteresis observed experimentally. There is slight inconsistency with the strength achieved by the wall from VecTor3 in comparison to the experimental observations from Menegon et al. (2017). For example, a maximum shear strength of 331 kN was determined by VecTor3 (monotonic loading), whereas a maximum of 318 kN was recorded experimentally. However, this is to be expected, as the “building core underwent a minor loss in its lateral strength equal to approximately 10% of its maximum capacity, occurring from a lateral drift value of 0.8% [21 mm] up to a value of 2.2% [57 mm]” (Menegon et al., 2017). This loss in strength was believed to be a result of localised failures from “patched” sections near the base of the wall.
Figure 10 Force-displacement results from VecTor3 for wall specimen S02

The curvature distribution results from VecTor3 (monotonic and reverse cyclic) are illustrated in Figure 11 for average drift values of 0.7% (18 mm) and 2.2% (57 mm). These curvatures were calculated using the strain values in the boundary regions of the wall (in the respective extreme fibre regions of the wall).

It is important that VecTor3 can accurately predict the curvature distribution up the wall height of lightly reinforced and unconfined C-shaped walls, as the curvatures will ultimately be used to calculate the equivalent plastic hinge lengths of the proposed walls in Section 3.3. Superimposed in Figure 11 are the curvature distributions reported in Menegon et al. (2017). The curvature distributions predicted by VecTor3 correlate well with the experimental observations. It should be noted that it was observed experimentally by Menegon et al. (2017) that the lap slice used in the design of specimen S02 caused two large cracks to form, which explains the two regions where the curvature distribution observed experimentally appear to be more concentrated. In contrast, the VecTor3 model, which does not model the lap splice and discontinuity of longitudinal reinforcement up the wall height, predicts a more evenly distributed curvature up the wall height. Moreover, the VecTor3 model predicts that yielding of the reinforcement only occurs in the lower 1200 mm of the wall, which correlates well with the experimental observations. Similar to the conclusions from the VecTor2 study in Hoult et al. (2017a), the strains (and
thus, curvatures) predicted by the model when reverse cyclic loading was imposed did not differ much from the predictions when the wall was subjected to monotonic loading.

Figure 11 Curvature distribution comparisons for (a) reverse cyclic loading and (b) monotonic loading

VecTor3 was able to predict the force-displacement relationship of a lightly reinforced and unconfined C-shaped wall. Importantly, the predicted cracking and curvature distribution up the height of the wall generally agreed with the experimental observations that were available.

3.3 Proposed C-shaped Walls for Low-to-moderate Seismic Regions

In total, 144 lightly reinforced and unconfined C-shaped walls with varying parameters have been analysed in VecTor3 for the calculation of the equivalent plastic hinge length and force-displacement relationship. Three different wall sizes have been chosen to represent a C-shaped RC core enclosing elevators for a low-rise (LR), mid-rise (MR) and high-rise (HR) building. The number of storeys of the LR, MR and HR C-shaped walls was chosen to be 3, 6 and 12 respectively, which are within the range given in FEMA (2010) and Maqsood et al. (2014). Using an inter-storey height of 3500 mm, the
effective height \((h_e \approx 0.7H_a)\) is approximately 7.35 m, 14.70 m and 29.40 m for the LR, MR and HR wall respectively. The number of elevator cars required for the different rise of building determines the size of the walls based on the recommendations given in RLB (2014). The LR C-shaped wall was assumed to enclose two elevator cars (2x500kg, 6 person), while the MR C-shaped wall was assumed to enclose three cars (3x900kg, 12 person) and the HR C-shaped wall was assumed to enclose four cars (4x1150kg, 16 person). The internal dimensions (width x depth, in mm) of the 500kg, 900kg and 1150kg elevator cars are 1000 x 1300, 1400 x 1500 and 1500 x 1800 respectively. Therefore, the resulting lengths of the web \((L_{\text{web}})\) and flange \((L_{\text{flange}})\) that were required for the three different walls are given in Table 6. After some discussions with P. McBean (personal communication, February 26, 2016), a consulting engineer and Joint Managing Director of Wallbridge & Gilbert, 200mm thick walls and a length of 600 mm (approximately 2 ft) for the returns \((L_{\text{return}})\) were taken as values that reflect current and past practice in Australia. The thickness \((t_w)\) of the HR C-shaped wall was increased to 250 mm, as shown in Table 6. An example of the cross-section for the LR C-shaped wall is given in Figure 12. Two elements are used across the thickness of the walls. Moreover, the 3:2 aspect ratio for plane stress elements as recommended in Wong and Vecchio (2002) and Palermo and Vecchio (2007) was adhered to, but further extrapolated to hexahedral elements for VectoR3. As was practiced in Hoult et al. (2017a), a refined mesh size was used in the predicted “yielding zone”, an approach that has been successfully employed by other researchers (Bohl & Adebar, 2011; Kwan, 1996). The element size in the x, y and z-directions \((s_x, s_y, s_z)\) respectively, the total number of elements \((n_e)\) and total number of nodes \((n_t)\) used in all three models are given in Table 7.

<table>
<thead>
<tr>
<th>Wall</th>
<th>(t_w) (mm)</th>
<th>(L_{\text{web}}) (mm)</th>
<th>(L_{\text{flange}}) (mm)</th>
<th>(L_{\text{return}}) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR</td>
<td>200</td>
<td>3600</td>
<td>2000</td>
<td>600</td>
</tr>
<tr>
<td>MR</td>
<td>200</td>
<td>6200</td>
<td>2200</td>
<td>600</td>
</tr>
<tr>
<td>HR</td>
<td>250</td>
<td>8500</td>
<td>2500</td>
<td>600</td>
</tr>
</tbody>
</table>
The material and constitutive models that were summarised in Section 3.1 are to be used in VecTor3 for these C-shaped walls. The range of parameters considered in VecTor3 are summarised in Table 8. The effective height ($H_e$) is taken as $0.7H_n$ as recommended by Priestley et al. (2007) for cantilever walls and all of the aspect ratios ($A_e$) of the walls are greater than 2. The $A_e$ of the C-shaped walls are different depending on the direction of loading; for bending about the major axis, the $A_e$ is taken as $H_e/L_{web}$, whereas for bending about the minor axis the $A_e$ is taken as $H_e/L_{flange}$. These two different $A_e$ values for each of the walls are given in Table 8. The $ALRs$ have been chosen to represent common values found in Australia and other low-to-moderate seismic regions (Albidah et al., 2013; Henry, 2013). The axial load was distributed to all nodes at the top of the wall and was held constant throughout the analyses. The minimum transverse reinforcement ratio of 0.25% from AS 3600:2009 (Standards Australia, 2009) is
used for all of the walls, while a range of longitudinal reinforcement ratios were used to represent
common values used in low-to-moderate seismic regions (Wibowo et al., 2013).

The same $f_{cmi}$ values used in Hoult et al. (2017a) for the VecTor2 analyses of rectangular walls are used
here; the two $f_{cmi}$ values considered in this study were 40 MPa and 60 MPa, which are realistic values for
walls designed originally with characteristic compressive strengths ($f_c$) of 32 MPa and 40 MPa
respectively. Mean values of the material properties for the reinforcing steel were taken from Menegon et
al. (2015) for D500N bars and are given in Table 9, which conform to AS/NZS 4671:2001 (Standards
Australia/New Zealand, 2001). The ultimate strain of the reinforcing steel was taken as 0.6$\varepsilon_{su}$ based on
the recommendations from Priestley et al. (2007) and as discussed previously in Section 3.2.1.

<table>
<thead>
<tr>
<th>Wall</th>
<th>$A_r$ (m$^2$)</th>
<th>$H_r$ (m)</th>
<th>$ALR$ (%)</th>
<th>$f_{cmi}$ (MPa)</th>
<th>$\rho_{ev}$ (%)</th>
<th>$\rho_{wh}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR</td>
<td>2.1$^a$, 3.7$^b$</td>
<td>7.35</td>
<td>1.5, 5</td>
<td>40, 60</td>
<td>0.15, 0.45, 0.70, 1.0</td>
<td>0.25</td>
</tr>
<tr>
<td>MR</td>
<td>2.4$^a$, 6.7$^b$</td>
<td>14.70</td>
<td>1.5, 5</td>
<td>40, 60</td>
<td>0.15, 0.45, 0.70, 1.0</td>
<td>0.25</td>
</tr>
<tr>
<td>HR</td>
<td>3.4$^a$, 11.7$^b$</td>
<td>29.30</td>
<td>1.5, 5</td>
<td>40, 60</td>
<td>0.15, 0.45, 0.70, 1.0</td>
<td>0.25</td>
</tr>
</tbody>
</table>

$^a$: bending about the major axis
$^b$: bending about the minor axis

Table 9: Mean values of the mechanical properties of D500N reinforcement from Menegon et al. (2015)

<table>
<thead>
<tr>
<th>Material</th>
<th>$f_y$ (MPa)</th>
<th>$f_u$ (MPa)</th>
<th>$f_u / f_y$</th>
<th>$\varepsilon_{th}$</th>
<th>$\varepsilon_{su}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>D500N</td>
<td>551</td>
<td>660.5</td>
<td>1.201</td>
<td>0.0197</td>
<td>0.095</td>
</tr>
</tbody>
</table>

Using the same approach that was adopted for analysing the C-shaped walls tested by Beyer et al. (2008b)
in Section 3.2.1, the nodes at the base of wall were assigned fixed conditions, while the nodes around the
outer faces of the wall had constraints such that the wall would not move out of plane. A 1 mm lateral
displacement was applied to all nodes at the top of the wall, and these displacements were monotonically
increased until “failure” of the wall. The three different directions in which the displacements were
applied at the top of the wall corresponded with the wall bending about the major axis and minor axis
(with WiC and WiT). Due to time constraints, it was not possible to conduct analyses which involved
cyclically varying displacements. It should be noted that while C-shaped walls can be subject to diagonal
loading, this research is limited to focusing on two primary modes of flexure: bending parallel to the web
and flanges. Furthermore, analyses that included “twisting” of the wall were not conducted; the author
acknowledges that in a “real world” situation, a C-shaped wall in a building is likely to twist due to
seismic loading being applied offset from the shear centre of the wall (particularly for bending about the
major axis), the primary focus of this investigation was for in-plane loading (with the 3 modes of
bending). Failure of the wall was deemed to occur when one of either two things occurred: (i) the
Collapse Prevention strain limit was reached in the concrete or steel, for which an unconfined concrete
strain value of 0.003 (0.3%) and steel strain value of 0.05 (5%) were used, or (ii) the maximum
displacement was reached corresponding to drift limits of 2.5% for the LR walls and 1.5% for the MR and
HR walls. It should be noted that the definition of drift used here corresponds to the displacement at roof
level relative to the height of the wall. In VecTor3 it was not possible to control the inter-storey drifts,
but simply the overall drift. In AS 1170.4:2007 (Standards Australia, 2007) the inter-storey drift
corresponding to the design level earthquake cannot exceed 1.5% of the storey height. It is expected that
at average drift levels of 1.5% or 2.5% for these walls, the inter-storey drift requirement from AS
1170.4:2007 will have been exceeded, so that the crack pattern would be reasonably representative of that
which could exist at the design level earthquake and there would not be any point in continuing the
analyses any further than this. As noted previously, the equivalent plastic hinge length will be
approximately constant once the cracking pattern is well established (Section 3.2.1). Hence, if the drift
limit was reached before the material strain limits, the longitudinal strain distribution corresponding to
this drift limit could be used to find a sufficiently accurate estimation of the equivalent plastic hinge
length.

4. Equivalent Plastic Hinge Length Results

For all of the walls analysed in VecTor3, curvature distributions up the height of the wall were obtained
by using the concrete and steel strains in the extreme fibre regions at the point at which the maximum
displacement was reached. The plastic hinge length \( L_p \) was calculated using the EPHL method discussed previously in Section 2.2. The following sections discuss the \( L_p \) values resulting from the VecTor3 analyses with consideration given to the different directions of loading.

4.1 Major Axis

The results for the \( L_p \) as a function of the \( \rho_{uv} \) range used for each of the walls and for bending about the major axis are shown in Figure 13(a) and Figure 13(b) for \( f_{cm} \) values of 40 MPa and 60 MPa respectively. Superimposed on these figures (red square-dot lines) are indications of the respective \( \rho_{uv,min} \) calculated from Equation 1. For all of the walls with a \( \rho_{uv} \) less than \( \rho_{uv,min} \), a small \( L_p \) can be observed in Figure 13, corresponding to strains being concentrated at a single, primary crack. This concentration of strain for walls with a \( \rho_{uv} \) less than \( \rho_{uv,min} \) was also observed for the rectangular walls analysed in VecTor2 in Hoult et al. (2017a). While the \( \rho_{uv,min} \) does give a good estimate of the reinforcement ratio required to trigger the onset of secondary cracking, there is not a correspondingly well-defined large increase in \( L_p \) for reinforcement ratios above this value, as there was for the rectangular walls in Hoult et al. (2017a).

![Figure 13 Equivalent plastic hinge lengths for the C-shaped walls and for bending about the major axis with (a) \( f_{cm} \) of 40 MPa and (b) \( f_{cm} \) of 60 MPa](image)

It was also found in Hoult et al. (2017a) that the concrete strains \( (\epsilon_c = 0.003) \) governed the performance of the rectangular walls for sections with \( \rho_{uv} \) larger than \( \rho_{uv,min} \), i.e. this limit was reached before the tensile strain limit. While the concrete strains are also found to govern the performance of the C-shaped walls...
for sections with ρ_{sv} larger than ρ_{sv, min} (and for bending about the major axis), in many of these walls the steel strains (in the extreme tension fibre) are relatively low (compared with those in the rectangular walls) when the ultimate concrete strain is reached. This is illustrated in Figure 14 for the MR C-shaped wall, with an ALR of 5% and a ρ_{sv} of 0.70%. While the ρ_{sv} of the wall was higher than the calculated ρ_{sv, min}, such that secondary cracking occurred, the displacement capacity was limited to just 40 mm (top wall displacement), or just 0.27% average drift, due to the ultimate unconfined concrete strain of 0.003 being reached in the boundary region of the wall. In the case of the MR wall in Figure 14, the length over which plasticity occurred in the extreme tension fibre region of the wall was small (L_p ≈ 256 mm), which was due to the premature compression failure hindering the potential for the plasticity to develop up the wall from the base. As the ρ_{sv} increases, the displacement capacity remains relatively constant due to the unconfined concrete strain governing, whereas the magnitude of the steel tension strains decrease. Thus, the equivalent plastic hinge length reduces as the ρ_{sv} is increased (and secondary cracking is achieved); this observation would not be expected in well confined walls that have the potential for a higher ultimate concrete strain.

For this direction of loading, VecTor3 predicts that the concrete strains are concentrated at the corners of the “boundary region” (Figure 14b), corresponding to the intersection of the web and flange, and not spread out evenly over the width of the flange, as is usually assumed in bending theory. The concentration of compression strains in the boundary regions of the wall, depending on the direction of loading, can be understood by the shear lag phenomenon (Kwan, 1996), where the Bernoulli-Euler assumption that plane sections remain plane after bending is only approximate. Further discussions on these results and effects from shear lag are given in Hoult et al. (2017b).
Figure 14 MR wall with strains in (a) tension (steel) (b) compression (concrete) at 40 mm top wall displacement

A total of 24 C-shaped walls had a $\rho_{\text{w}}$ higher than the $\rho_{\text{w,min}}$ and for bending about the major axis. Four key parameters were scrutinised in potentially influencing the $L_p$, the first three of these key parameters, wall length ($L_w$), axial load ratio ($ALR$) and effective height ($H_e$), have previously been observed to influence the spread of plasticity in the wall (Hoult et al., 2017a; Kazaz, 2013). The fourth parameter was the normalised average shear stress, which was found to influence the $L_p$ for the C-shaped walls analysed numerically in Constantin (2016). Statistical analyses and regression analyses were performed using XLSTAT (Addinsoft, 2014). The correlation matrix (Pearson correlation coefficient) is given in Table 10 for all four variables and the equivalent plastic hinge length ($L_p$) results from VecTor3 for bending about the major axis; correlation coefficients will vary from -1 to 1, where negative values indicate a negative correlation and vice versa. Furthermore, correlation values close to zero indicate the absence of correlation. Table 10 shows that all four variables had some influence on the plastic hinge length results.
from VecTor3. The wall length ($L_w$) provides the largest positive correlation to $L_p$, while the normalised shear stress parameter ($\psi$) provided the highest negative correlation to $L_p$.

### Table 10 Correlation matrix (Pearson correlation coefficient) for the $L_p$ and bending about major axis

<table>
<thead>
<tr>
<th>Variables</th>
<th>$L_w$</th>
<th>$H_e$</th>
<th>$ALR$</th>
<th>$\psi = \frac{\tau}{0.17(f_c')^{(1/2)}}$</th>
<th>$L_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_w$</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_e$</td>
<td>0.97</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$ALR$</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\psi = \frac{\tau}{0.17(f_c')^{(1/2)}}$</td>
<td>-0.88</td>
<td>-0.86</td>
<td>0.36</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>$L_p$</td>
<td>0.55</td>
<td>0.54</td>
<td>-0.57</td>
<td>-0.76</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Plotting the calculated $L_p$ against the four significant parameters indicate that the $L_w$, $H_e$, and $ALR$ have linear trends, while the normalised shear parameter ($\psi$) is nonlinear. Therefore, a multiple nonlinear regression analysis was used to derive the plastic hinge length expression for unconfined lightly reinforced concrete C-shaped walls and for bending about the major axis (Equation 9).

$$L_p = (0.1L_w - 0.013H_e)(1 - 13ALR)(7e^{-0.8\psi}) \leq 0.5L_w$$

A maximum limit of 0.5$L_w$ is used in Equation 9; this is the same as that proposed by many other researchers (Moehle, 1992; Paulay, 1986; Priestley & Park, 1984; Wallace & Moehle, 1992).

### 4.2 Minor Axis (WiC)

The results for the $L_p$ as a function of the $\rho_{w}$ range used for each of the walls and for bending about the minor axis (WiC) are shown in Figure 15(a) and Figure 15(b) for $f_{cm}$ values of 40 MPa and 60 MPa respectively. The $\rho_{w,\text{min}}$ calculated from the secondary cracking model (SCM) appears to accurately indicate the amount of reinforcement required for secondary cracking. These figures are similar to the performance observed for the rectangular RC walls in Hoult et al. (2017a), where the walls with a $\rho_{w}$ less than $\rho_{w,\text{min}}$ generally result in a small $L_p$. In contrast, the walls with a $\rho_{w}$ greater than $\rho_{w,\text{min}}$ result in a greater $L_p$ value. Moreover, as these RC walls were governed by tension strains, a much larger $L_p$ can be
observed for this direction of loading in comparison to the $L_f$ achieved for bending about the major axis.

An example of the concrete and steel longitudinal strain distributions across the cross-section for one of the analysed LR C-shaped walls ($f_{cm}$ of 40 MPa, ALR of 5% and $\rho_{wv}$ of 0.70%), which reached the average drift limit of 2.5% before the strain limits were reached, is given in Figure 16. The shear lag phenomenon is clearly observed in this figure, with concentrations of tension and compression strains in the corners of the boundary ends.

![Graphs showing the distribution of strains](image)

**Figure 15** Equivalent plastic hinge lengths for the C-shaped walls and for bending about the minor axis (WiC) with (a) $f_{cm}$ of 40 MPa and (b) $f_{cm}$ of 60 MPa

![Steel and concrete strain distributions](image)

**Figure 16** (a) steel tensile and (b) concrete compressive strains across the cross-section of the LR wall
Statistical and regression analyses were performed again using XLSTAT (Addinsoft, 2014) with the results for \(L_p\) for bending about the minor axis (WiC). The correlation matrix given in Table 11 shows that all four variables had some influence on the plastic hinge length results from VecTor3.

<table>
<thead>
<tr>
<th>Variables</th>
<th>(L_w)</th>
<th>(H_e)</th>
<th>(ALR)</th>
<th>(v = \pi/0.17(f'_c)^{1/2})</th>
<th>(L_p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L_w)</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(H_e)</td>
<td>0.99</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ALR)</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(v = \pi/0.17(f'_c)^{1/2})</td>
<td>-0.89</td>
<td>-0.86</td>
<td>0.23</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>(L_p)</td>
<td>0.71</td>
<td>0.71</td>
<td>-0.24</td>
<td>-0.67</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Similar to the previous section for the walls bending about the major axis, a multiple nonlinear regression analysis was used to derive the plastic hinge length expression for unconfined lightly reinforced concrete C-shaped walls and for bending about the minor axis (WiC) (Equation 10).

\[
L_p = (0.5L_w - 0.015H_e)(1 - 3ALR)(1.6e^{-0.1v}) \leq L_w
\]

### 4.3 Minor Axis (WiT)

The results for the \(L_p\) as a function of the \(\rho_{sv}\) range used for each of the walls and for bending about the minor axis (WiT) are shown in Figure 17(a) and Figure 17(b) for \(f_{cm}\) values of 40 MPa and 60 MPa respectively. As expected, for the walls with a \(\rho_{sv}\) less than the indicated \(\rho_{sv,min}\), a small \(L_p\) can be observed in Figure 17. While the \(\rho_{sv,min}\) does provide a good estimate for the onset of secondary cracking, for the majority of the results the \(L_p\) does not increase with larger reinforcement ratios, something that was also observed for some of the C-shaped wall analyses for bending about the major axis. This is due to the concrete strains governing the performance of the wall; the ultimate unconfined concrete strain (\(\varepsilon_c = 0.3\%\)) is reached either before the tensile strains in the extreme tension fibre reach yield, or, if yield is reached, the spread of plasticity up the wall is limited. Of the 24 walls that were deemed to have a satisfactory amount of longitudinal reinforcement to allow secondary cracking, only 11 reached the yield
strain in tension ($\varepsilon_y \approx 0.27\%$) prior to the ultimate unconfined concrete strain in the concrete being reached. Therefore, the majority of these walls, while cracked, remained essentially elastic in flexural behaviour up until failure of the wall due to the compression strain limit being reached in the unconfined boundary ends of the flanges. However, the authors acknowledge there will be some compression softening (and therefore, some inelastic behaviour) caused by the concrete reaching and exceeding its compression capacity. An example of this is illustrated in Figure 18 for the IIR wall ($f_{cm}^t$ of 40 MPa, $ALR$ of 1.5%, $\rho_{sw}$ of 1.00%), where the maximum steel and concrete strains observed at a top wall displacement of 251 mm are 0.18% and 0.28% respectively.

Figure 17 Equivalent plastic hinge lengths for the C-shaped walls and for bending about the minor axis (WiT) with (a) $f_{cm}^t$ of 40 MPa and (b) $f_{cm}^t$ of 60 MPa
Figure 18 HR wall with strains in (a) tension (steel) (b) compression (concrete) at 251 mm top wall displacement

Nevertheless, the ability of the walls to undergo inelastic rotations before the concrete crushes is dependent on the $ALR$, $\rho_{w}$, and $f_{cm}$ values. Therefore, an equivalent plastic hinge length equation was derived using the results for these walls. The correlation results from the statistical analysis performed using XLSTAT (Addinsoft, 2014) is shown in Table 12, which indicate that the $L_{w}$ and $H_{c}$ provide little correlation to the calculated $L_{p}$ from the VecTor3 results for bending about the minor axis (WiT).

However, it was found that the $ALR$ and normalised shear stress ($v$) parameters are negatively correlated to the resulting $L_{p}$ for these types of walls and mode of bending.
Table 12 Correlation matrix for the $L_p$ and bending about minor axis (WiC)

<table>
<thead>
<tr>
<th>Variables</th>
<th>$L_w$</th>
<th>$H_e$</th>
<th>$ALR$</th>
<th>$v = \pi/0.17(f'_c)^{(1/2)}$</th>
<th>$L_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_w$</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_e$</td>
<td>1.00</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$ALR$</td>
<td>0.12</td>
<td>0.12</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v = \pi/0.17(f'_c)^{(1/2)}$</td>
<td>-0.92</td>
<td>-0.91</td>
<td>0.10</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>$L_p$</td>
<td>0.03</td>
<td>0.00</td>
<td>-0.54</td>
<td>-0.31</td>
<td>1.00</td>
</tr>
</tbody>
</table>

A multiple nonlinear regression analysis was used to derive the plastic hinge length expression for unconfined lightly reinforced concrete C-shaped walls and for bending about the minor axis with the web in tension (WiT) (Equation 11). While it could be argued that this equation could be more concise, given that it was found that two of the parameters used here provided very little correlation to the $L_p$ (Table 12), Equation 11 was derived and kept in the same format as those derived for bending about the major axis (Equation 9) and minor axis with WiC (Equation 10) for consistency. Furthermore, a slightly stronger $R^2$ correlation was achieved by including all of these parameters in a multiple nonlinear regression analysis in comparison to the linear regression analysis that only used $ALR$ and $v$.

$$L_p = (1.0L_w - 0.073H_e)(1 - 8ALR)(2.5e^{-2.1v}) \leq 0.5L_w$$

5. Summary of $L_p$ Expressions

The expressions for lightly reinforced and unconfined C-shaped walls that have been derived from the analyses undertaken in this study, and that were found to form secondary cracks, are summarised below:

For bending about the major axis:

$$L_p = (0.1L_w - 0.013H_e)(1 - 13ALR)(7e^{-0.8v}) \leq 0.5L_w$$

For bending about the minor axis (WiC):

$$L_p = (0.5L_w - 0.015H_e)(1 - 3ALR)(1.6e^{-0.1v}) \leq L_w$$
For bending about the minor axis (WiT)

\[ L_p = (1.0L_w - 0.073H_e)(1 - 8ALR)(2.5e^{-2.1\nu}) \leq 0.5L_w \]

All of the VecTor3 results for \( L_p \) (for walls with a \( \rho_{wv} \) greater than \( \rho_{wv,mn} \)) are plotted in Figure 19 in comparison to the \( L_p \) estimates from the expressions introduced in Section 2.2 (Equations 5 through 7).

These expressions in the literature produce a poor estimate of the \( L_p \) in comparison to results from VecTor3 for \( L_p \) of lightly reinforced and unconfined C-shaped RC walls. In contrast, the expressions derived from the study here (above) give the strongest correlation, shown in Figure 20. This is not all that surprising, since the expressions from this study were derived from the VecTor3 data.

![Graphs showing correlation]

**Figure 19** \( L_p \) results from VecTor3 compared to estimates using expressions from (a) Priestley *et al.* (2007) (b) Hoult *et al.* (2017a) and (c) Constantin (2016)
Figure 20 $L_p$ estimation from expressions derived here in comparison to the VecTor3 results

Each of the expressions above follow the same format but with quite different constants for each of the parameters, which reflect the different failure modes observed for the different modes of bending. For example, the $ALR$ was found to be less correlated with the $L_p$ results for the C-shaped walls with bending about the minor axis (WiC) (Table 11), which were governed by tension strains, in comparison to the walls that were governed by compression strains when bending about the major and minor (WiT) axes (Table 10 and Table 12, respectively). Therefore, the $L_p$ expression above for bending about the minor axis (WiC) has a smaller, negative constant for the $ALR$ in comparison to the same constants used in the expressions above for bending about the major and minor (WiT) axes.

Figure 21 compares each of the expressions above with some of the $L_p$ expressions available in the literature (discussed in Section 2) for a case study wall. The $L_{web}$, $L_{flange}$ and $L_{return}$ of this C-shaped wall were assumed to be 5000 mm, 2000 mm and 600 mm, while the $\rho_{sw}$ was 1.00%. The $H_r$, $f_{cm}$ and $ALR$ were assumed to be 15.68 m, 40 MPa and 2.5% respectively. Moment-curvature analyses were required to calculate the normalised shear stress parameter needed for several of these $L_p$ expressions. The moment-curvature analyses using RESPONSE-2000 (Bentz, 2000) estimated ultimate moments of 33786 kNm, 8544 kNm and 15676 kNm for bending about the major, minor axis (WiC) and minor axis (WiT) respectively. For this C-shaped wall, Figure 21 indicates that the $L_p$ expression derived in this study for bending about the minor axis (WiC) is similar to the value obtained using other available expressions.
This is not all that surprising, given that the C-shaped walls analysed in this study were found to perform generally well for this direction of loading, reaching large displacement capacities and allowing tension strains to distribute up a significant portion of the wall height. In contrast, the premature compression failure hindered the potential for plasticity to develop up the wall from the base for the C-shaped walls analysed in this study and for bending about the major and minor (WiT) axes. Thus, the predicted $L_p$ from the expressions derived in this study are much lower than the values obtained using some of the other expressions available in the literature, which reflects the vulnerable nature of unconfined C-shaped walls.

![](image)

**Figure 21** Comparison of plastic hinge length for case study wall

6. Conclusions

The aim of this paper was to investigate the equivalent plastic hinge length of lightly reinforced and unconfined C-shaped walls that are a popular construction choice in Australia and other low-to-moderate seismic regions. As a paucity of experimental evidence exists for the seismic performance of such structural elements, this was deemed to be an important undertaking.

It has been shown that some of the empirical plastic hinge length equations that are commonly used in a plastic hinge analysis are inappropriate for these types of walls and for this type of detailing. In some of
the cases considered here, the unconfined boundary regions of the C-shaped walls were found to play a significant role in preventing a good distribution of plasticity on the tension side, since the ultimate unconfined concrete strain was reached in these regions (on the compressive side) at very low displacement capacities.

Equivalent plastic hinge length \( L_p \) equations were derived for lightly reinforced and unconfined C-shaped walls using numerical analyses in VecTor3. Importantly, these equations are dependent on the different directions of loading; about the major axis, minor axis with web in compression (WiC) and with web in tension (WiT). There are some limitations on the use of the relationships that have been derived in the preceding sections for the \( L_p \). This is due to the limited range of the parameters used to derive these expressions. For example, the \( L_p \) equations derived from the VecTor3 results are specific to unconfined C-shaped RC walls with uniformly distributed longitudinal reinforcement. The axial load ratio and the longitudinal reinforcement ratio are limited to a maximum of 5% and 1% respectively, since these are the maximum values used in this study. The aspect ratio of the walls should be equal to or greater than 2.

The \( f_y/f_c \) ratio will also have an influence on the spread of plasticity. Therefore, the derived \( L_p \) equations are also limited to the assessment of walls with N-type reinforcement, and more conservative equations would be required for design. However, it should be noted it is expected that these expressions would give conservative estimates of the \( L_p \) if a more ductile reinforcement, such as D500E, were used. It is also important to note that these expressions have been derived from the VecTor3 results, which do not consider the yield penetration into the foundation; while the foundation was not modelled in VecTor3, previous research found that the VecTor finite element modelling software struggles to capture the strain penetration into the foundation using truss-link elements (Hoult et al., 2017a). Therefore, these equations proposed here could result in a conservative value, given that the yield strain penetration length into the foundation was not included in the \( L_p \) expressions resulting from these analyses. However, separate expressions for the yield strain penetration length into the foundation from Priestley et al. (2007) (Equation 6) or Altheeb et al. (2015) could be considered.
The numerical analyses conducted in VecTor3 for this research investigation has highlighted the vulnerability of lightly reinforced and unconfined C-shaped walls. As previously mentioned throughout this paper, there has been very little experimental testing that has focused on the seismic performance of non-rectangular (e.g. C-shaped) RC walls with poor detailing that is commonly found in Australia and other low-to-moderate seismic regions. It is therefore emphasised as a priority for an experimental program be undertaken to evaluate the seismic performance of RC C-shaped walls with this type of detailing; that is, C-shaped walls that are lightly reinforced and unconfined in the boundary regions. These experimental results will further support the results of the numerical investigation undertaken here.

The author intends to address the important issues related to the displacement capacity of these types of walls, as well as the rectangular walls investigated in Hoult et al. (2017a), in a separate paper, which will compare the results of the extensive numerical analyses, some of which have been conducted for the research investigated here, to the commonly used expressions used in a plastic hinge analysis.

This research has highlighted the vulnerability of lightly reinforced and unconfined C-shaped walls in Australia and other low-to-moderate seismic regions.

7. Acknowledgments

The support of the Commonwealth of Australia through the Cooperative Research Centre program is acknowledged.

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