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A Note on the Maximum Severity of Ruin and Related Problems

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Abstract

Picard (1994) defines the maximum severity of ruin, M_u, to be the largest deficit of a classical surplus process, starting from initial surplus u, between the time of ruin and the time of recovery to surplus level 0. He gives a simple expression for the distribution function of M_u in terms of the probability of ultimate ruin. This paper first addresses the question of calculating the moments of M_u. It is not easy to achieve explicit expressions for these despite knowing the distribution function of M_u. We consider situations where explicit expressions can be obtained, as well as approximations. We also consider the closely related question of the maximum surplus prior to ruin.

1 Introduction

We consider the classical surplus process $\{U(t)\}_{t\geq 0}$ defined as

$$U(t) = u + ct - \sum_{i=1}^{N(t)} X_i$$

where u is the insurer's initial surplus, c is the rate of premium income per unit time, $N(t)$ is the number of claims up to time t, and X_i is the amount of the ith claim. $\{N(t)\}_{t\geq 0}$ is a Poisson process, whose parameter we will always set to 1. $\{X_i\}_{i=1}^{\infty}$ is a sequence of i.i.d. random variables, independent of $\{N(t)\}_{t\geq 0}$. We denote by μ_k the kth moment of X_i, and without loss of generality we set $\mu_1 = 1$. Let F and f denote the distribution function and density function respectively of X_i, with $F(0) = 0$. Finally, we write $c = 1 + \theta$, where $\theta > 0$ is the premium loading factor. We define the distribution function F_1 by

$$F_1(x) = \int_0^x (1 - F(y))dy$$

for $x > 0$, with density function f_1.

When the moment generating function of X_i exists, the adjustment coefficient, denoted R, is the unique positive number such that

$$1 + (1+\theta)R = E\left[\exp\{RX_i\}\right].$$

We say that ruin occurs if the surplus falls below 0, and we define the time of ruin, T, as

$$T = \begin{cases} \inf(t\colon\ U(t) < 0) \\ \infty \text{ if } U(t) \geq 0 \text{ for all } t > 0. \end{cases}$$

The probability of ultimate ruin from initial surplus u is denoted $\psi(u)$ and defined as $\psi(u) = \Pr(T < \infty)$, with $\delta(u) = 1 - \psi(u)$.

Given that ruin occurs, we define T' to be the time of the first upcrossing of the surplus process through 0 after time T. For finite T, Picard (1994) defines

$$M_u = \sup\{|U(t)|,\ T \leq t \leq T'\}$$

where the subscript u denotes initial surplus. Let

$$J_u(z) = \Pr(M_u \leq z | U(0) = u \ \text{ and } \ T < \infty).$$

Picard shows that for $z \geq 0$

$$J_u(z) = \frac{\psi(u) - \psi(u+z)}{\psi(u)(1-\psi(z))}. \tag{1.1}$$

Thus, if we know the function ψ, we know J_u. Picard also shows that the probability that the maximum deficit occurs at ruin, given that ruin occurs, is given by

$$\int_0^\infty \tilde{g}(u,y)\frac{\delta(0)}{\delta(y)}dy \tag{1.2}$$

where $\tilde{g}$ is the density of the deficit at ruin, given that ruin occurs. We note that $\tilde{g}(0,y) = f_1(y)$ (see, for example, Bowers *et al* (1998)).

In this paper we consider the moments of M_u. Throughout we consider only the first two moments, but the ideas generalise to higher moments. In Section 2 we look at two cases when explicit expressions for the moments of M_u can be found, and we illustrate approximations based on these expressions in Section 3. The related problem of the maximum surplus prior to ruin is discussed in Section 4, and the question of whether the maximum surplus prior to ruin occurs immediately prior to ruin is the topic of Section 5

2 Explicit solutions

To find explicit formulae for the moments of M_u we need an explicit solution for ψ. We will illustrate two situations in which we can find expressions for the moments of M_u. These expressions are based on two types of formula for ψ.

2.1 Case (i)

Let us suppose that $F(x) = 1 - \exp\{-x\}$, $x > 0$, so that

$$\psi(u) = \bar{R}\exp\{-Ru\}, \tag{2.1}$$

where $R = \theta/(1+\theta)$ and $\bar{R} = 1 - R$. See, for example, Bowers *et al* (1998). Then

$$\begin{aligned} J_u(z) &= \frac{1 - e^{-Rz}}{1 - \bar{R}e^{-Rz}} \\ &= (1 - e^{-Rz})\sum_{j=0}^{\infty} \bar{R}^j e^{-Rjz} \\ &= \sum_{j=1}^{\infty} w_j(1 - e^{-Rjz}) \end{aligned}$$

where $w_j = R\bar{R}^{j-1}$, so that $\sum_{j=1}^{\infty} w_j = 1$. Thus, the distribution of M_u is an infinite mixture of exponential distributions. This representation allows us to write down expressions for all the moments of M_u. In particular, the first two moments of M_u are

$$E(M_u) = (1+\theta)\log(1+\theta^{-1})$$

and

$$E(M_u^2) = \frac{2(1+\theta)^2}{\theta}\sum_{j=1}^{\infty}\frac{(1+\theta)^{-j}}{j^2}$$

(as given in Dickson and Egídio dos Reis (1997).) Thus, the moments of M_u depend only on the loading factor θ. They are independent of u since the distribution of the deficit at ruin is independent of u - see, for example, Bowers *et al* (1998). Even in this case - the most straightforward one - we need to calculate the second moment via an infinite series. Table 2.1 shows values of the mean and standard deviation of M_u for different values of θ. As we would expect, these quantities decrease as θ increases.

θ	$E(M_u)$	$s.d.(M_u)$
0.05	3.197	7.324
0.1	2.638	5.007
0.15	2.342	4.015
0.2	2.150	3.443
0.25	2.012	3.064
0.3	1.906	2.792

Table 2.1: Values of $E(M_u)$ and $s.d.(M_u)$ - exponential claims

2.2 Case (ii)

Now suppose that the ruin probability is of the form

$$\psi(u) = ae^{-Ru} + be^{-Tu},$$

where $R, T > 0$ and a and b are constants such that $a + b = \psi(0)$. Such a form arises if the individual claim amount distribution is an Erlang(2) distribution or a mixture of two exponential distributions. In this case we can find expressions for the first two moments of M_u from

$$E(M_u) = \int_0^\infty (1 - J_u(z))dz \tag{2.2}$$

and

$$E(M_u^2) = 2\int_0^\infty z(1 - J_u(z))dz. \tag{2.3}$$

We can write

$$\begin{aligned} 1 - J_u(z) &= \frac{1}{1-\psi(z)}\left(\frac{\psi(u+z)}{\psi(u)} - \psi(z)\right) \\ &= \frac{1}{1-\psi(z)}\left(a_u e^{-Rz} + b_u e^{-Tz} - \psi(z)\right) \end{aligned}$$

where $a_u = ae^{-Ru}/\psi(u)$, and similarly for b_u. Hence

$$1 - J_u(z) = \left((a_u - a)e^{-Rz} + (b_u - b)e^{-Tz}\right)\sum_{r=0}^\infty \psi(z)^r,$$

and so

$$E(M_u) = (a_u - a)\sum_{r=0}^\infty \int_0^\infty e^{-Rz}\psi(z)^r dz + (b_u - b)\sum_{r=0}^\infty \int_0^\infty e^{-Tz}\psi(z)^r dz$$

$$
\begin{aligned}
&= (a_u - a)\sum_{r=0}^{\infty}\int_0^{\infty} e^{-Rz}\sum_{j=0}^{r}\binom{r}{j} a^j e^{-jRz} b^{r-j} e^{-(r-j)Tz} dz \\
&\quad +(b_u - b)\sum_{r=0}^{\infty}\int_0^{\infty} e^{-Tz}\sum_{j=0}^{r}\binom{r}{j} a^j e^{-jRz} b^{r-j} e^{-(r-j)Tz} dz \\
&= (a_u - a)\sum_{r=0}^{\infty}\sum_{j=0}^{r}\binom{r}{j} a^j b^{r-j}\int_0^{\infty} e^{-((j+1)R+(r-j)T)z} dz \\
&\quad +(b_u - b)\sum_{r=0}^{\infty}\sum_{j=0}^{r}\binom{r}{j} a^j b^{r-j}\int_0^{\infty} e^{-(jR+(r-j+1)T)z} dz \\
&= (a_u - a)\sum_{r=0}^{\infty}\sum_{j=0}^{r}\binom{r}{j} a^j b^{r-j}\frac{1}{(j+1)R+(r-j)T} \\
&\quad +(b_u - b)\sum_{r=0}^{\infty}\sum_{j=0}^{r}\binom{r}{j} a^j b^{r-j}\frac{1}{jR+(r-j+1)T} \qquad (2.4)
\end{aligned}
$$

Similarly, we find that

$$
\begin{aligned}
E(M_u^2) &= 2(a_u - a)\sum_{r=0}^{\infty}\int_0^{\infty} ze^{-Rz}\psi(z)^r dz + 2(b_u - b)\sum_{r=0}^{\infty}\int_0^{\infty} ze^{-Tz}\psi(z)^r dz \\
&= 2(a_u - a)\sum_{r=0}^{\infty}\sum_{j=0}^{r}\binom{r}{j} a^j b^{r-j}\frac{1}{((j+1)R+(r-j)T)^2} \\
&\quad +2(b_u - b)\sum_{r=0}^{\infty}\sum_{j=0}^{r}\binom{r}{j} a^j b^{r-j}\frac{1}{(jR+(r-j+1)T)^2}. \qquad (2.5)
\end{aligned}
$$

These expressions are also in terms of infinite series, but we can nevertheless evaluate them, as terms in the summations in (2.4) and (2.5) go to zero as r increases, and so we can truncate the summations.

Example 2.1 *Suppose that the individual claim amount distribution is Erlang(2). Table 2.2 shows values of the mean and standard deviation of M_u for some values of u and θ. We see that for each value of θ, the mean and standard deviation of M_u decrease rapidly to the limiting values as $u \to \infty$. This is explained by the behaviour of the conditional distribution of the severity of ruin as a function of u. (See Egídio dos Reis (1993).) As in Table 2.1 we see that as θ increases, both $E(M_u)$ and s.d.(M_u) decrease for a given value of u.*

We note that a sufficient condition for the moments of M_u to exist is that the adjustment coefficient exists. To see this we note that

$$
J_u(z) = \frac{\psi(u) - \psi(u+z)}{\psi(u)(1-\psi(z))} \geq 1 - \frac{\psi(u+z)}{\psi(u)}
$$

	$\theta = 0.1$		$\theta = 0.2$		$\theta = 0.3$	
u	$E(M_u)$	$s.d.(M_u)$	$E(M_u)$	$s.d.(M_u)$	$E(M_u)$	$s.d.(M_u)$
0	2.025	3.726	1.652	2.544	1.464	2.050
1	1.825	3.553	1.484	2.428	1.311	1.957
2	1.813	3.542	1.473	2.420	1.300	1.950
3	1.813	3.542	1.473	2.420	1.299	1.949
4	1.813	3.542	1.473	2.420	1.299	1.949
5	1.813	3.542	1.473	2.420	1.299	1.949

Table 2.2: Values of $E(M_u)$ and $s.d.(M_u)$ - Erlang(2) claims

so that

$$1 - J_u(z) \leq \frac{\psi(u+z)}{\psi(u)} \leq \frac{e^{-R(u+z)}}{\psi(u)}$$

and hence

$$\begin{aligned} E(M_u^r) &= r\int_0^\infty z^{r-1}\left(1 - J_u(z)\right) dz \\ &\leq \frac{re^{-Ru}}{\psi(u)} \int_0^\infty z^{r-1} e^{-Rz} dz \\ &= \frac{e^{-Ru}}{\psi(u)} \frac{\Gamma(r+1)}{R^r} < \infty. \end{aligned}$$

3 Approximations

In this section we consider two approximation methods based on the results of the previous section.

3.1 De Vylder's Approximation

De Vylder (1978) considers the situation when the first three moments of the individual claim amount distribution exist. His procedure involves approximating our surplus process by a classical surplus process with Poisson parameter $\lambda = 9\mu_2^3/2\mu_3^2$, individual claim amount distribution

$$\tilde{F}(x) = 1 - \exp\{-\alpha x\}$$

where $\alpha = 3\mu_2/\mu_3$, and rate of premium income per unit time $\tilde{c} = \theta + 3\mu_2^2/2\mu_3$, leading to the approximation:

$$\tilde{\psi}(u) \approx \frac{\lambda}{\alpha\tilde{c}} \exp\left\{-(\alpha - \lambda/\tilde{c})u\right\}.$$

Then by applying the techniques of Section 2.1 it follows that we can approximate $E(M_u)$ by

$$\frac{\tilde{c}}{\lambda}\log\left(\frac{\alpha\tilde{c}}{\alpha\tilde{c}-\lambda}\right)$$

and $E(M_u^2)$ by

$$\frac{2\tilde{c}^2}{\lambda(\alpha\tilde{c}-\lambda)}\sum_{j=1}^{\infty}\frac{(\lambda/\alpha\tilde{c})^j}{j^2}.$$

3.2 Cramer's Asymptotic Formula

Cramer's asymptotic formula gives rise to the approximation

$$\psi(u) \approx Ce^{-Ru}$$

where $C = \theta/(E[X_i \exp\{RX_i\}] - 1 - \theta)$. See, for example, Gerber (1979). It follows that if we use this expression for ψ, we can approximate $E(M_u)$ by

$$\frac{1-C}{CR}\log(1-C)^{-1}$$

and $E(M_u^2)$ by

$$\frac{2(1-C)}{R^2}\sum_{j=1}^{\infty}\frac{C^{j-1}}{j^2}.$$

We note that an obvious disadvantage of these approximation procedures is that the approximations are independent of u. However, as we see from Example 2.1, the moments can be sensitive to the value of u only over a small range. Note that the use of Cramer's formula as an approximation to ψ requires the existence of the adjustment coefficient, whereas De Vylder's approximation does not. However, numerical illustrations in De Vylder's paper suggest that his approximation works best when the adjustment coefficient exists.

Example 3.1 *Consider the same set-up as in Example 2.1. Table 3.1 shows approximations and exact values of the mean and standard deviation of M_5 for different values of θ. We observe that the approximations are reasonable in each case. There is little difference between the standard deviations, but the values of the mean are understated using the Cramer approximation, and overstated using De Vylder's.*

	Exact values		Approx. - De Vylder		Approx. - Cramer	
	$E(M_5)$	$s.d.(M_5)$	$E(M_5)$	$s.d.(M_5)$	$E(M_5)$	$s.d.(M_5)$
$\theta = 0.1$	1.813	3.542	1.819	3.561	1.805	3.545
$\theta = 0.2$	1.473	2.420	1.485	2.443	1.465	2.423
$\theta = 0.3$	1.299	1.949	1.316	1.976	1.291	1.952

Table 3.1: Approximations to $E(M_5)$ and $s.d.(M_5)$ - Erlang(2) claims

θ	C	R	T
0.1	0.7734	0.0036	0.0917
0.2	0.6209	0.0059	0.1028
0.3	0.5147	0.0074	0.1126

Table 3.2: Parameters for Tijms' approximations

3.3 Tijms' Approximation

Tijms (1986) proposes the following approximation to ψ:

$$\psi(u) \approx Ce^{-Ru} + \left(\frac{1}{1+\theta} - C\right) e^{-Tu}$$

where C and R are as previously defined, and T is such that the mean of the compound geometric distribution, for which ψ gives the tail probability, is preserved under the approximation. This approximation is exact if the individual claim amount distribution is Erlang(2) or a mixture of two exponential distributions. Using this approximation to ψ, we can apply formulae (2.4) and (2.5).

Example 3.2 *Let*

$$F(x) = \sum_{i=1}^{3} \alpha_i (1 - \exp\{-\beta_i x\}), \quad x > 0,$$

with $\alpha_1 = 0.0039793$, $\alpha_2 = 0.1078392$, $\alpha_3 = 0.8881815$, $\beta_1 = 0.014631$, $\beta_2 = 0.190206$ *and* $\beta_3 = 5.51451$. *Wikstad (1971) cites this distribution as a model for individual claims based on Swedish fire insurance data. Table 3.2 shows parameters for Tijms' approximations to* ψ *for three different values of* θ*, Table 3.3 shows some approximations to, and exact values of,* $E(M_u)$*, and Table 3.4 shows approximations to, and exact values of, the standard deviation of* M_u *for some values of* u.

The exact values were obtained by numerical integration using (2.2) and (2.3) and explicit solutions for ψ*. In principle, this approach could be used in any*

	$\theta = 0.1$		$\theta = 0.2$		$\theta = 0.3$	
u	Approx.	Exact	Approx.	Exact	Approx.	Exact
0	44.79	44.51	36.75	36.50	33.05	32.82
10	85.71	86.59	71.69	72.18	65.65	65.89
20	105.06	104.00	88.63	87.46	81.70	80.40
30	113.55	112.39	95.76	94.65	88.13	87.03
40	117.15	116.33	98.57	97.85	90.49	89.83
50	118.66	118.15	99.66	99.24	91.33	90.98

Table 3.3: Approximate and exact values of $E(M_u)$

	$\theta = 0.1$		$\theta = 0.2$		$\theta = 0.3$	
u	Approx.	Exact	Approx.	Exact	Approx.	Exact
0	117.38	117.50	86.87	86.99	74.80	74.93
10	158.02	158.26	117.03	116.95	101.18	100.94
20	170.57	169.80	125.70	125.05	108.35	107.74
30	175.13	174.48	128.50	128.05	110.44	110.08
40	176.90	176.50	129.49	129.24	111.11	110.93
50	177.62	177.39	129.85	129.73	111.33	111.26

Table 3.4: Approximate and exact values of $s.d.(M_u)$

situation in which we have a formula or numerical values for ψ. We note that the approximations are reasonably good in both Table 3.3 and Table 3.4. We also note that in this example, both $E(M_u)$ and $s.d.(M_u)$ vary considerably with u, unlike in Table 2.2. This suggests that the approximations of Sections 3.1 and 3.2 would not be appropriate for this individual claim amount distribution.

4 The maximum surplus prior to ruin

Let us define N_u to be the maximum value of the surplus process prior to ruin, given that ruin occurs. For finite T we define

$$N_u = \sup\{U(t),\ \ 0 < t \leq T\},$$

where the subscript u again denotes initial surplus.

The probability that ruin occurs from initial surplus u without reaching surplus level $z > u$ prior to ruin is

$$\frac{\psi(u) - \psi(z)}{1 - \psi(z)},$$

and the probability that the surplus process attains z without ruin occurring is

$$\chi(u,z) = \frac{\delta(u)}{\delta(z)}.$$

(See Dickson and Gray (1984).) Hence, for $z \geq u$,

$$K_u(z) = \Pr(N_u \leq z | U(0) = u \text{ and } T < \infty) = \frac{\psi(u) - \psi(z)}{\psi(u)(1 - \psi(z))}.$$

Notice that when $u = 0$, we have $K_0(z) = J_0(z)$, a result which can be explained by dual events (see, for example, Dickson(1992)).

Example 4.1 *Let $F(x) = 1 - \exp\{-x\}$. Then using results from Section 2.1 we have*

$$\begin{aligned} K_u(z) &= \frac{1 - e^{-R(z-u)}}{1 - \bar{R}e^{-Rz}} \\ &= \left(1 - e^{-R(z-u)}\right) \sum_{j=0}^{\infty} \left(\bar{R}e^{-Rz}\right)^j. \end{aligned}$$

In the special case when $u = 0$, we have

$$K_0(z) = \sum_{j=1}^{\infty} w_j (1 - e^{-Rjz})$$

where $w_j = R\bar{R}^{j-1}$. Thus, $K_0(z) = J_u(z)$ for all $u \geq 0$ since $J_u(z)$ is independent of u for this claim amount distribution.

We can find the mean of N_u as

$$\begin{aligned} E(N_u) &= u + \int_u^{\infty} (1 - K_u(z))\, dz \\ &= u + (e^{Ru} - \bar{R}) \int_u^{\infty} \frac{e^{-Rz}}{1 - \bar{R}e^{-Rz}} dz \end{aligned}$$

Writing

$$\left(1 - \bar{R}e^{-Rz}\right)^{-1} = \sum_{j=0}^{\infty} \bar{R}^j e^{-Rjz}$$

we find that

$$E(N_u) = u + \frac{e^{Ru} - \bar{R}}{R\bar{R}} \log\left(1 - \bar{R}e^{-Ru}\right)^{-1}.$$

Similarly,

$$E(N_u^2) = u^2 + \frac{2(1 - \bar{R}e^{-Ru})}{R^2} \sum_{j=0}^{\infty} \frac{(\bar{R}e^{-Ru})^j}{(j+1)^2} \left[1 + R(j+1)u\right].$$

u	$\theta = 0.1$		$\theta = 0.2$		$\theta = 0.3$	
	$E(N_u)$	$s.d.(N_u)$	$E(N_u)$	$s.d.(N_u)$	$E(N_u)$	$s.d.(N_u)$
0	2.638	5.007	2.150	3.443	1.906	2.792
1	4.991	6.356	4.062	4.201	3.606	3.320
10	18.68	9.744	15.50	5.743	14.16	4.247
100	111.0	11.00	106.0	6.000	104.3	4.333
1,000	1,011	11.00	1,006	6.000	1,004	4.333

Table 4.1: Values of $E(N_u)$ and $s.d.(N_u)$ - exponential claims

Table 4.1 shows some values of the mean and standard deviation of N_u. As expected, the moments of N_u depend on u, in contrast to the moments of M_u. We note that for each value of θ, the mean and standard deviation of $N_u - u$ both tend to $1/R$ as $u \to \infty$. A feature of Table 4.1 is that both the mean and standard deviation of N_u decrease as θ increases. This is intuitively reasonable, as if ruin occurs with a large value of θ, it is likely to occur quickly. It is also consistent with the result that $E(T|T < \infty)$ is a decreasing function of θ - see Gerber (1979).

The ideas from Section 2.2 can also be applied to find the moments of N_u when

$$\psi(u) = ae^{-Ru} + be^{-Tu}.$$

We find that

$$\begin{aligned} 1 - K_u(z) &= \left(\psi(u)^{-1} - 1\right) \sum_{r=1}^{\infty} \psi(z)^r \\ &= \left(\psi(u)^{-1} - 1\right) \sum_{r=1}^{\infty} \sum_{j=0}^{r} \binom{r}{j} a^j b^{r-j} e^{-(Rj+T(r-j))z} \end{aligned}$$

giving

$$E(N_u) = u + \left(\psi(u)^{-1} - 1\right) \sum_{r=1}^{\infty} \sum_{j=0}^{r} \binom{r}{j} a^j b^{r-j} \frac{e^{-(Rj+T(r-j))u}}{Rj + T(r-j)}.$$

We can again calculate moments noting that terms in the above summation become very small for large values of r. Similarly we can calculate $E(N_u^2)$.

In Section 2, we showed that a sufficient condition for the moments of M_u to exist is that the adjustment coefficient exists. The same is true for the moments of N_u when $u < \infty$. To see this, we first note that for $z \geq u$,

$$\left(1 - e^{-Rz}\right)^{-1} \leq \left(1 - e^{-Ru}\right)^{-1}$$

and that

$$1 - K_u(z) = \frac{1-\psi(u)}{\psi(u)} \frac{\psi(z)}{1-\psi(z)}. \tag{4.1}$$

We then have

$$\begin{aligned} E(N_u^r) &= u^r + r\int_u^\infty z^{r-1}\left(1-K_u(z)\right)dz \\ &\le u^r + r\frac{1-\psi(u)}{\psi(u)}\int_u^\infty z^{r-1}\frac{\psi(z)}{1-\psi(z)}dz \\ &\le u^r + r\frac{1-\psi(u)}{\psi(u)}\int_u^\infty z^{r-1}\frac{e^{-Rz}}{1-e^{-Ru}}dz \\ &\le u^r + \frac{1-\psi(u)}{\psi(u)}\frac{1}{1-e^{-Ru}}\frac{\Gamma(r+1)}{R^r} < \infty. \end{aligned}$$

We noted in Example 4.1 that the mean and standard deviation of $N_u - u$ both tend to $1/R$ as $u \to \infty$. This feature is not just restricted to the case of exponential claims. If the adjustment coefficient exists, Cramer's asymptotic formula is $\psi(u) \sim Ce^{-Ru}$. Writing

$$E(N_u) = u + \int_u^\infty \left(1-K_u(z)\right)dz$$

and using (4.1) we have

$$E(N_u - u) = \frac{1-\psi(u)}{\psi(u)}\int_u^\infty \frac{\psi(z)}{1-\psi(z)}dz.$$

Then

$$\begin{aligned} \lim_{u\to\infty} E(N_u - u) &= \lim_{u\to\infty}\frac{1}{\psi(u)}\int_u^\infty \frac{\psi(z)}{1-\psi(z)}dz \\ &= \lim_{u\to\infty}\frac{1}{\psi'(u)}\frac{-\psi(u)}{1-\psi(u)} \\ &= \lim_{u\to\infty}\frac{-\psi(u)}{\psi'(u)} \\ &= 1/R. \end{aligned}$$

A similar argument shows that $\lim_{u\to\infty} V(N_u - u) = 1/R^2$.

5 Does the maximum surplus before ruin occur immediately prior to ruin?

In this section we consider whether the surplus immediately prior to ruin is the maximum surplus prior to ruin. We can approach this problem using

dual events. Consider the following two events.
Event 1: ruin occurs from initial surplus 0, with a deficit of $y > u > 0$ at ruin, with y being the maximum deficit before recovery to surplus level 0.
Event 2: ruin occurs from initial surplus 0 with a crossing through the surplus level u prior to ruin, and with the maximum surplus before ruin occurring immediately prior to ruin.

If we consider a realisation of the surplus process satisfying the conditions of Event 1, we can construct a dual process $\{U^*(t)\}$ defined by

$$\begin{aligned} U^*(t) &= -U(T' - t) \quad \text{for } 0 \le t \le T', \\ U^*(t) &= U(t) \quad \text{for } t > T'. \end{aligned}$$

Then for any realisation of the surplus process satisfying the conditions of Event 1, there is a unique realisation of the dual process which satisfies the conditions of Event 2, and which has the same probability density.

Figure 1 shows a realisation of the surplus process which satisfies the conditions of Event 1 with $u = 1$ and $y = 1.5$, and Figure 2 shows the corresponding dual realisation.

Define $\phi(u)$ to be the probability that ruin occurs from initial surplus u, with the maximum surplus before ruin occurring immediately prior to ruin. Equating the probabilities of Events 1 and 2 we have

$$\psi(0) \int_u^\infty f_1(y)\chi(0,y)dy = \chi(0,u)\phi(u)$$

giving

$$\phi(u) = \psi(0)\delta(u) \int_u^\infty \frac{f_1(y)}{\delta(y)} dy. \tag{5.1}$$

We note that in the special case when $u = 0$, equation (5.1) yields

$$\frac{\phi(0)}{\psi(0)} = \int_0^\infty \tilde{g}(0,y) \frac{\delta(0)}{\delta(y)} dy$$

consistent with expression (1.2) for the probability that the maximum deficit occurs at ruin, given that ruin occurs.

Example 5.1 *Let $F(x) = 1 - \exp\{-x\}$, with ψ given by (2.1). Then*

$$\begin{aligned} \phi(u) &= \bar{R}\left(1 - \bar{R}e^{-Ru}\right) \int_u^\infty \frac{e^{-y}}{1 - \bar{R}e^{-Ry}} dy \\ &= \bar{R}\left(1 - \bar{R}e^{-Ru}\right) \int_u^\infty e^{-y} \sum_{j=0}^\infty \left(\bar{R}e^{-Ry}\right)^j dy \end{aligned}$$

	$\theta = 0.1$	$\theta = 0.2$	$\theta = 0.3$
u	$\phi(u)/\psi(u)$	$\phi(u)/\psi(u)$	$\phi(u)/\psi(u)$
0	0.6243	0.6490	0.6709
1	0.3029	0.3374	0.3693
2	0.1326	0.1590	0.1851
3	0.0561	0.0724	0.0897
4	0.0233	0.0324	0.0427
5	0.0096	0.0144	0.0202

Table 5.1: Values of $\phi(u)/\psi(u)$ - exponential claims

	$\theta = 0.1$	$\theta = 0.2$	$\theta = 0.3$
u	$\psi_1(u)/\psi(u)$	$\psi_1(u)/\psi(u)$	$\psi_1(u)/\psi(u)$
0	0.5238	0.5455	0.5652
1	0.2110	0.2371	0.2619
2	0.0850	0.1030	0.1214
3	0.0343	0.0448	0.0562
4	0.0138	0.0195	0.0261
5	0.0056	0.0085	0.0121

Table 5.2: Values of $\psi_1(u)/\psi(u)$ - exponential claims

$$= \bar{R}\left(1 - \bar{R}e^{-Ru}\right)\sum_{j=0}^{\infty}\bar{R}^j\int_u^{\infty}e^{-(Rj+1)y}dy$$

$$= \bar{R}\left(1 - \bar{R}e^{-Ru}\right)\sum_{j=0}^{\infty}\frac{\bar{R}^j}{Rj+1}e^{-(Rj+1)u}$$

Table 5.1 shows values of $\phi(u)/\psi(u)$ for some values of u and θ. We observe from Table 5.1 that if ruin occurs, the higher the initial surplus, the less likely it is that the maximum of the surplus process occurs immediately before ruin. We can also see that as θ increases, the more likely it is that the maximum surplus occurs immediately before ruin for a fixed value of u.
For this model, the probability of ruin at the first claim is

$$\psi_1(u) = \int_0^{\infty} e^{-\tau}e^{-u-(1+\theta)\tau}d\tau = \frac{e^{-u}}{2+\theta}.$$

Table 5.2 shows values of $\psi_1(u)/\psi(u)$ for the same values of u and θ as in Table 5.1. From this table we see that if the initial surplus is small and if ruin occurs with the maximum surplus before ruin occurring immediately prior to ruin, then there is a high probability that ruin occurred at the first claim.

	$\theta = 0.1$		$\theta = 0.25$	
u	Lower	Upper	Lower	Upper
100	0.0456	0.0546	0.1437	0.1517
1,000	0.0791	0.0801	0.1897	0.1905
10,000	0.0893	0.0894	0.1986	0.1987
100,000	0.0907	0.0907	0.1998	0.1998

Table 5.3: Bounds for $\phi(u)/\psi(u)$

More generally, we can find bounds for ϕ. Since $\delta(u) \leq \delta(y) \leq 1$ for $u \leq y$, it follows from (5.1) that

$$\psi(0)\delta(u)(1 - F_1(u)) \leq \phi(u) \leq \psi(0)(1 - F_1(u)).$$

Thus, if we can calculate ψ, we can easily calculate bounds for ϕ, or for ϕ/ψ. We observe that this bound should be tight for values of u which are large relative to the mean individual claim amount.

Example 5.2 *Suppose now that $F(x) = 1-(1+x)^{-2}$. Table 5.3 shows some bounds for $\phi(u)/\psi(u)$ for a range of values of u. In calculating these values, we have used values of ψ given by Usábel (2001). This table suggests the following:*

(i) for a given value of θ, $\phi(u)/\psi(u)$ increases with u, and

(ii) for a given value of u, $\phi(u)/\psi(u)$ increases with θ.

The second of these observations is in line with the findings in Table 5.1, but the first is not, suggesting that the behaviour of $\phi(u)/\psi(u)$ depends on the tail behaviour of the individual claim amount distribution. The values of u in Table 5.3 are very large, and, if ruin occurs, we would expect it to occur when a very large claim occurs. For the values of u in Table 5.3, the probability of ruin at the first claim is negligible. For example, when $u = 100$ and $\theta = 0.1$, we have

$$\psi_1(u) = \int_0^\infty \frac{e^{-\tau}}{(1 + u + (1 + \theta)\tau)^2} d\tau = 9.6 \times 10^{-5},$$

so that $\psi_1(u)/\psi(u) = 5.8 \times 10^{-3}$, compared with $\phi(u)/\psi(u)$ lying in the interval from 0.0456 to 0.0546.

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Figure 1: a realisation of the surplus process satisfying the conditions of Event 1

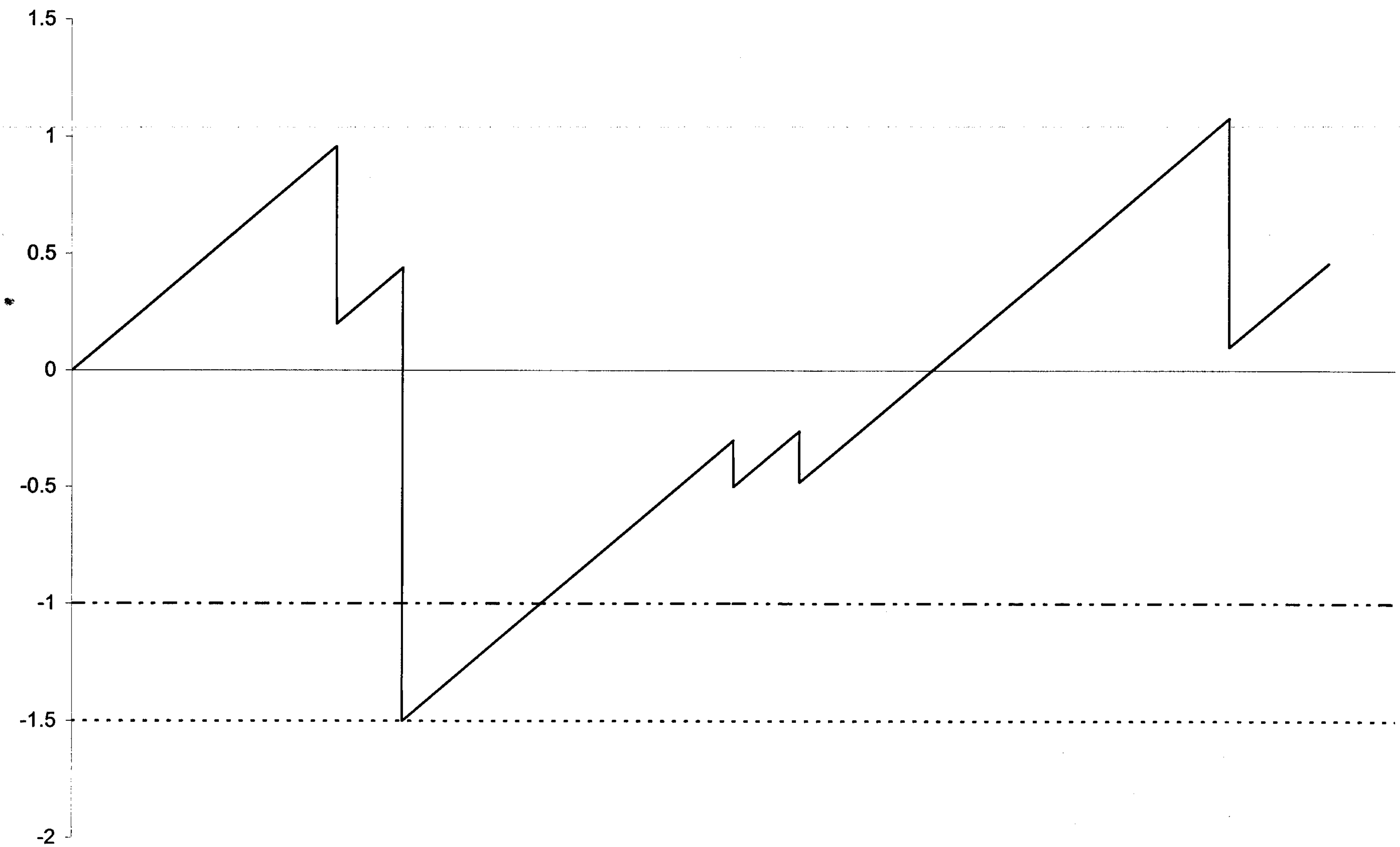
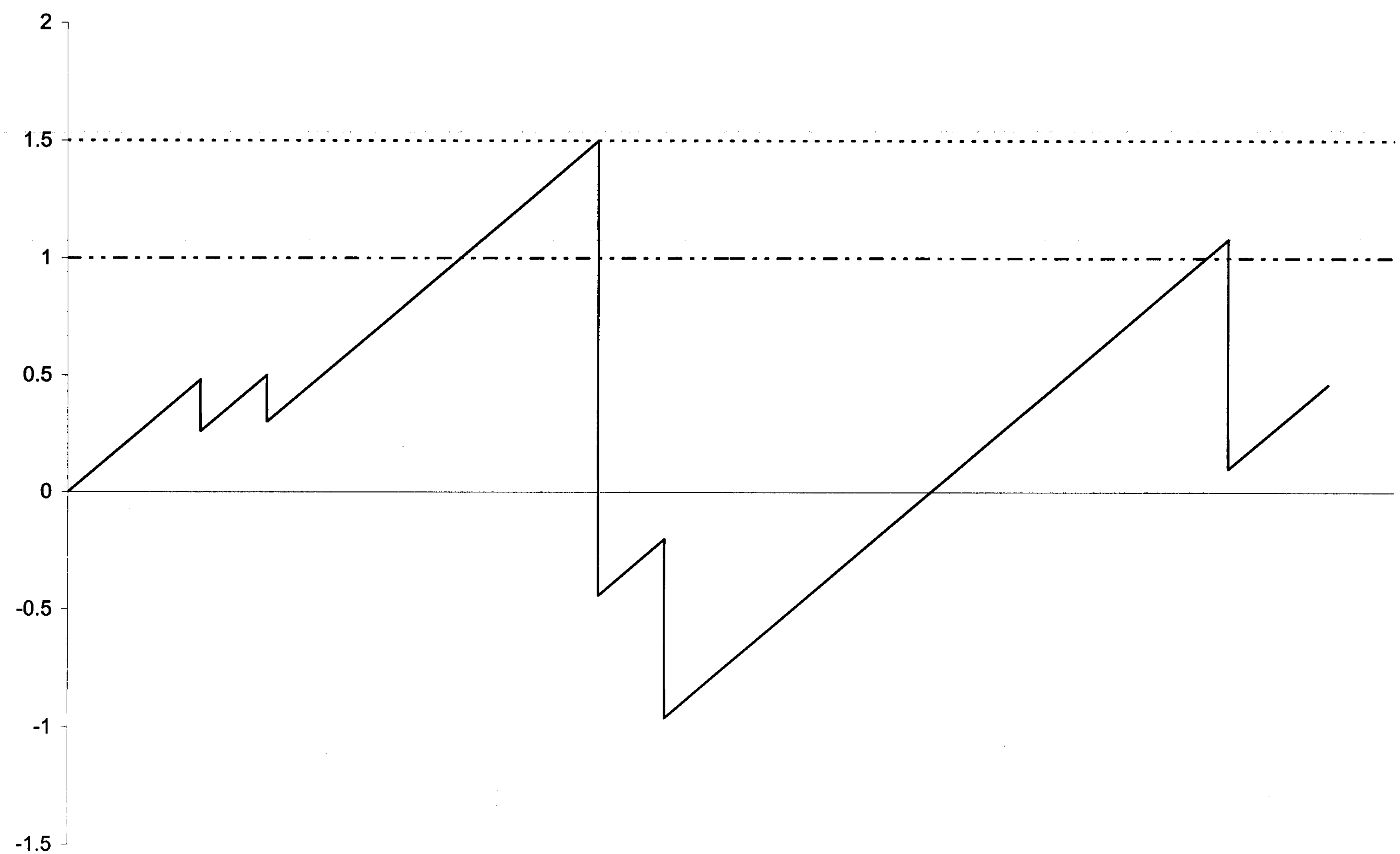


Figure 2: the dual realisation corresponding to Figure 1



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