Heat Transfer Coefficient Estimation for Turbulent Boundary Layers

S. Wang¹, Y. Xia¹, W. Abu Rowin⁴, I. Marusic¹, R. Sandberg⁴, D. Chung¹, N. Hutchins¹, K. Tanimoto² and T. Oda²

¹ Department of Mechanical Engineering
University of Melbourne, Victoria 3010, Australia
² Research Innovation Center
Takasago, Mitsubishi Heavy Industries Ltd., Japan

Abstract
Convective heat transfer in rough wall-bounded turbulent flows is prevalent in many engineering applications, such as in gas turbines and heat exchangers. At present, engineers lack the design tools to accurately predict the convective heat transfer in the presence of non-smooth boundaries. Accordingly, a new turbulent boundary layer facility has been commissioned, where the temperature of an interchangeable test surface can be precisely controlled, and conductive heat losses are minimized. Using this facility, we can estimate the heat transfer coefficient (Stanton number, St), through measurement of the power supplied to the electrical heaters and also from measurements of the thermal and momentum boundary layers evolving over this surface. These methods have been initially investigated over a shorter smooth prototype heated surface and compared with existing St prediction models. Preliminary results suggest that we can accurately estimate St in this facility.

Keywords
Heat transfer; Boundary layers; Heat transfer coefficient

Introduction
When a moving fluid comes into contact with a stationary solid surface and where a temperature difference exists between the fluid and solid, convective heat transfer occurs. The heat transfer coefficient is defined by the Stanton number (St), which can be considered as the heat transfer analogue to the skin-friction coefficient (Cf). Kays & Crawford [3] and White & Corfield [11] derived the St equation from an analogue to the Clauser chart based on the inner scaled mean temperature profile. Kays & Crawford [3] also derived a relationship between St and enthalpy thickness based on the energy integral equation. Ligrani & Moffat [5, 7] estimated St from an energy balance performed on each segment of an electrically heated plate. Ligrani & Moffat [5] used a power meter to measure the total electrical power, estimating convective heat flux as the total electrical power minus the heat losses due to conductive and radiative heat transfer. Mukerji et al. [7] used calorimeters to measure the wall heat flux directly.

The above-cited studies developed expressions to estimate St for turbulent boundary layers over heated smooth surfaces. However, in many engineering applications, the surfaces are non-smooth, which, for some of the above methods, adds complexity for estimating St. To study the influence of surface roughness on heat transfer, a turbulent boundary layer flow facility is under development with a controllable constant temperature test surface (that can be either smooth or rough). Here we describe the design of the facility and test several techniques to compute the local St over a heated smooth surface to validate the design and proposed instrumentation.

Estimating heat transfer coefficient
We consider three methods to estimate the local Stanton number (St). The first method computes St from the following expression using a direct measurement of the thermal and momentum boundary layers

\[
St = \frac{\Theta_t}{\Theta_w - \Theta_{\infty}} \sqrt{\frac{C_f}{2}},
\] (1)

where, \(\Theta_t\) is the wall temperature, \(\Theta_w\) is the free stream temperature and \(\Theta_f\) is the friction temperature estimated from the Clauser fit to the thermal boundary layer (discussed in the results section below). The skin friction coefficient \(C_f\) in equation (1) is evaluated from

\[
C_f = 2 \left( \frac{U_t}{U_\infty} \right)^2,
\] (2)

where \(U_t\) is the friction velocity, estimated from a Clauser fit to the momentum boundary layer, and \(U_\infty\) is the freestream velocity. There are concerns over the accuracy of this approach, especially for rough walls, where the Clauser fit involves additional fitting parameters (the Hama roughness function \(\Delta U^+\), its thermal analogue \(\Delta \Theta^+\) and also a virtual wall-normal origin) which increases the error. Note from equation (1), that this estimate for St will have compounded errors due to the application of two modified Clauser fits. For the present low Reynolds number conditions, we estimate that the error in St calculated via Clauser-type fits to equation (1) will be ±9%.

The second method is through measurement of the convective heat flux, \(q''_c\). The relationship between the Stanton number and forced convective heat flux is

\[
St = \frac{q''_c}{\rho C_p U_\infty (\Theta_w - \Theta_{\infty})}.
\] (3)

Here, \(\rho\) is the working fluid density and \(C_p\) is the fluid specific heat at constant pressure.

The rate of change of the enthalpy thickness along the streamwise direction, \(x\), can also be used to compute St. The derivation here (as described in [3, 11]) is similar to von Kármán’s mean momentum balance which for zero pressure gradient boundary layers yields the result \(C_f/2 = d\Theta/dx\) (\(\Theta\) is the momentum thickness and \(x\) is the streamwise direction). Through integrating the applicable form of the momentum and energy equations for an isothermal surface [3], the Stanton number is given by

\[
St = \frac{\delta_n}{\delta_w} = \int_0^\infty \frac{U}{U_\infty} \frac{\Theta - \Theta_{\infty}}{\Theta_{\infty} - \Theta_{\infty}} \, dx
\] (4)

Here, \(U\) and \(\Theta\) are the mean streamwise velocity and temperature respectively (which for given operational conditions will be a function of \(z, x\) and \(z\) is the wall-normal distance). This method is not assessed for the experiments reported here over a heated plate of limited streamwise fetch as it requires temperature measurement at several streamwise locations.
Test section design
Experiments are conducted in a boundary-layer wind tunnel (illustrated in figure 1a) of working section 5.7 × 0.94 × 0.38 m³ in x × y × z directions. For more details about the facility, the reader is referred to [9]. Measurements are made in the turbulent boundary layer developed over the tunnel floor. To provide a temperature difference between the solid boundary and the fluid, a new floor has been designed consisting of eleven 720 × 500 mm² (x × y) heated 6.35mm thick anodized smooth ALCA5 aluminum cast plates (see figure 1). Each plate is heated by a custom made heater pad (Holroyd Components Ltd) with a power of 1.22 kW each (3.4kW/m²). The floor temperature is monitored through thermocouples with 0.05 °C uncertainty embedded in the aluminum plates. Power to each heater is controlled individually with solid-state relays and a proportional–integral–derivative (PID) controller system to maintain the floor at a constant temperature. The power consumption for each heater is recorded with a Powertech MS-6108 power meter with 41W measurement uncertainty. Ideally, the majority of this measured power is transferred to the fluid via convective heat transfer, for which the design needs to minimise conductive heat loss. The heat loss through the floor of the tunnel is minimized with a 53 mm thick MetecnoPanel rigid polyisocyanurate foam insulation plate (0.02W/mK) and an additional 3 mm thick calcium-magnesium silicate thermal insulation sheet (0.032W/mK). Additional thermocouples on the bottom outer surface of the wind-tunnel are used to calculate the conduction heat loss from the bottom of the heated plates. As suggested by Hosni et al. [2], the conductive heat loss to the tunnel side walls is also reduced with a series of heated side-rails that are maintained at the same temperature as the test plates. An exploded view of the heated test-surface is provided in figure 1(b).

Initial experiments over prototype surface
All measurements presented here are conducted in zero-pressure-gradient turbulent boundary layers formed over a (heated and unheated) smooth surface at a streamwise distance x = 800 mm from the tripped inlet of the working section. It is emphasized that this is a test of the prototype surface, consisting of just two test plates covering the first meter downstream of the tripped inlet to the working section. For this test there were no heated side-rails, and the under-insulation consisted of 3 mm thick calcium-magnesium silicate type insulation material, hence the conductive heat losses were much larger in this case. This prototype tested surface is indicated by the yellow shaded regions in figure 1. Three sensors, installed on a traverse system, are utilized to measure velocity and temperature along the wall-normal direction as displayed in figure 1(a).

Table 1. Parameters of unheated and heated smooth surfaces.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Power (W)</th>
<th>U∞ (m/s)</th>
<th>Θ∞ (°C)</th>
<th>Θw (°C)</th>
<th>z (μm)</th>
<th>Uτ (m/s)</th>
<th>θτ (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit</td>
<td>(mg)</td>
<td>(°C)</td>
<td>(°C)</td>
<td>(mm)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uncertainty</td>
<td>41</td>
<td>0.025</td>
<td>0.05</td>
<td>0.2</td>
<td>5</td>
<td>0.048</td>
<td>0.059</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.025</td>
<td>0.05</td>
<td>0.2</td>
<td>5</td>
<td>0.048</td>
<td>0.059</td>
</tr>
</tbody>
</table>

Table 2. Uncertainty of the measurement.

Results
Initial experiments with the PID controller active, used a thermal imaging system and multiple distributed embedded and calibrated thermocouples to ensure that the prototype heated plate design and controller could maintain a constant wall temperature over the range of flow conditions, with no apparent discontinuity at the interface between plates. Figure 2(a) shows the mean streamwise velocity profiles, $U^+ = U/U_τ$, of the unheated surface from both the hot-wire and Pitot-tube, with good overall agreement. Here $U_τ$ is estimated from a Clauser chart fit with the logarithmic profile of $U^+ = k^{-1} \ln(z^+) + 4.17$, where
\( \kappa = 0.384 \) is the von Kármán constant. The velocity profiles also overlap with the reference profiles of the numerical simulation of Schlatter & Orlu [10] at a comparable \( \text{Re} = 974 \). The hot-wire and Pitot-tube measured velocity profiles are compared for the heated plate in figure 2(b). The Pitot-tube measured profile retains good agreement with the numerical profile defined as [11]

\[
\Theta^+ = \frac{1}{\kappa_0} \ln(z^+) + A_f(Pr),
\]

here \( \kappa_0 = \kappa / Pr_t = 0.47 \) is the slope coefficient (where \( Pr_t \) is the turbulent Prandtl number assumed to be 0.85 [11]) and \( A_f = 4.2 \) for air (\( Pr = 0.71 \) [11]). The error in this fit is estimated at \( \pm 7\% \) based on uncertainties in \( \kappa_0, A_f \) and the log linear fit.

The hot-wire voltage can be corrected by following the procedure suggested by Brun [1]

\[
E^2_{corr} = E^2 \left[ 1 - 0.5 \frac{\Theta - \Theta_{wire}}{\Theta_{wire} - \Theta_{ref}} \right]
\]

where, \( E_{corr} \) is the correct hot-wire voltage, \( E \) is the original hot-wire voltage and \( \Theta_{wire} \) is the hot-wire temperature estimated by the sensor temperature coefficient of resistance and the overheat ratio [1]. The corrected HWA measured mean velocity profile for the heated case is shown by the light blue symbols in figure 2(b), with much improved overall agreement with the Pitot-static and DNS results. The discrepancy of \( U^+ \) at \( z^+ < 40 \) could be related to the limited resolution of the thermocouple measurement near the wall, and will be improved in the future through use of cold-wires, which offer much finer wall-normal resolution. Nevertheless, figure 2 gives confidence that the mean velocity profiles measured by HWA in thermal boundary layers can be corrected from the mean temperature profile, and also that the momentum boundary layer in this case (where the stability parameter is close to zero), can be measured over the unheated plate with little loss of accuracy.

The stability parameter can be calculated from the preliminary measurements to confirm that buoyancy effects are small. We use a Monin-Obukhov stability parameter \( \delta_f / L \), where \( \delta_f \) is the thermal boundary layer thickness determined from figure 2(c), and \( L \) is the Obukhov length, which is calculated by

\[
L = \frac{-U^+_2 \Theta_{ref}}{\kappa g \rho c'}
\]

where, \( g \) is the gravitational acceleration. For the expected range of experimental conditions (\( Pr = 0.71 \)), the largest stability parameter occurs at the edge of the thermal layer where \( \delta_f / L \) = 0.0018 which is much smaller that the generally accepted limits that indicate a negligible buoyancy effect. Krug et al. [4] have recently suggested that the aspect ratio of the hierarchical turbulent structure can be extremely sensitive to the stability parameter. A full heating power, the proposed test facility will be able to produce weakly unstable boundary layers with a stability parameter as high as 0.02, which Krug et al. [4] suggest will alter the aspect ratio of turbulent structures from 14:1 (streamwise:wall-normal) to as low as 8:3:1, suggesting that this facility can be used to systematically investigate this effect.

Evaluating the Stanton number using equation (1) requires an estimate for the skin friction coefficient \( C_f \) which is calculated using equation (2), where \( U_2 \) is given by a Clauser fit to the mean velocity profile. The evolution of \( C_f \) evaluated in this manner is found to be in good agreement with that shown by Nagib & Chauhan [8]. The resulting estimate for St is shown by the black symbol on figure 3. We can also estimate St from the convective heat flux of equation (3). Based on the uncertainty shown in table 2, the error in the Stanton number assessed by the error propagation method is estimated as \( \pm 16\% \) for the measured power method and \( \pm 9\% \) for the Clauser method, as shown by the red and black error bars respectively in figure 3. As explained by Moffat et al. [6], the convective heat flux, \( q'_c \), is...
measurements are performed over a prototype version of the design, with limited streamwise fetch and without the heated side rails and underfloor insulation. For these experiments we used a traversable sensor package consisting a Pitot tube, hot-wire and a thermocouple to measure mean velocity and temperature profiles. Initial results confirm that for the operating conditions tested (excess wall temperature $\Theta_w - \Theta_\infty = 15\degree C$, $U_\infty = 20\, m/s$), this is an almost purely convective regime, with no discernable buoyancy effects (the heated fluid acts as a passive scalar). Results also indicate that the hot-wire measurement drift due to the thermal boundary layer can be corrected for the mean velocity profile using the mean temperature profile. The facility was validated by computing the heat transfer coefficient over a heated smooth surface with two techniques and compared with existing predictive models for the evolution of $St$. The results were consistent with the predictive models suggesting that, with the further improvements offered by the full-scale system, this facility can be used for investigations of forced convective heat transfer over rough and smooth surfaces.

Acknowledgements

The authors acknowledge support from the Australian Research Council via grants LP180100712 and DP200100969.

References


Conclusion

A modular system for a heated test surface has been described, which permits investigation of forced convective heat transfer in smooth- and rough-wall bounded turbulent flows. The proposed design provides accurate isothermal wall conditions, and minimises losses due to conduction. Preliminary validating measurements are performed over a prototype version of the system.
Minerva Access is the Institutional Repository of The University of Melbourne

Author/s:
Wang, S; Xia, Y; Abu Rowin, W; Marusic, I; Sandberg, R; Chung, D; Hutchins, N; Tanimoto, K; Oda, T

Title:
Heat Transfer Coefficient Estimation for Turbulent Boundary Layers

Date:
2020-12-11

Citation:

Persistent Link:
http://hdl.handle.net/11343/288707

License:
CC BY-NC