Drop sizes and population balance model for a Karr column

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Abstract

Drop sizes were investigated and measured in a Karr column. A stagewise population balance model was presented considering drop breakage and coalescence. The same breakage mechanisms observed in the single drop study were also found in column. The breakage probability model was selected by comparing the model from single drop study and the widely-used models. Model parameters were calculated based on a two-step optimization method. The results showed that the drop sizes decreased with an increase in reciprocating intensity, but changed slightly with phase velocities. The predicted drop sizes were compared to the experimental data in previous studies and good agreement was achieved. Finally, the population balance model along with some existing correlations were compared with the experimental data. It was found that the population balance model and correlations with the low agitation term provided better predictions of drop sizes in the Karr column.

Topic Heading and Keywords

Transport Phenomena and Fluid Mechanics:

Karr column, drop size distribution, population balance model, parameter optimization

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Developed in 1959, the Karr reciprocating plate column has found wide application in liquid-liquid extraction, especially in the purification of pharmaceutical compounds. The advantages of the Karr column compared to others columns include higher extraction efficiencies and throughput, lower axial mixing, and uniform shear such that they are especially suitable for systems that tend to emulsify.

The Karr column consists of a stack of perforated plates moving up and down periodically that are driven by a motor. Typically, the dispersed phase enters the column via a distributor and produces an initial drop size distribution. As the drops pass through the column, repeated coalescence and breakage occur due to the reciprocating motion of the plates, which results in changes in the drop size distribution. The knowledge of the drop sizes is of fundamental importance in the design of a Karr column. A smaller drop size provides a larger interfacial area available for mass transfer and enhances the process. However, if the drops become too small, the operating limit is reached earlier and is defined as the flooding point. It is therefore important to understand and predict the drop sizes in the Karr column for better operation and high-quality products. Over the past decades, studies on drop size distribution have been carried out in the Karr column at different scales with various systems. The drop size distribution in a Karr column is usually represented by the Sauter mean diameter ($d_{32}$).

Earlier studies focused on measuring drop size distribution and developing correlation for $d_{32}$ based on experimental results. Baird and Lane studied drop sizes in a 5.08 cm diameter Karr column with four liquid-liquid systems without mass transfer. The drops were captured near the bottom of the column to ensure they attained a steady size distribution. The $d_{32}$ was
found depend on the reciprocating intensities (plate amplitude × frequency) and the system properties, but was not significantly influenced by the throughput. Then by adding a gravitational term for low agitation to the turbulent dispersion model, the drop sizes correlated well with the experimental measurements. However, in the study of Boyadzhiev and Spassove\textsuperscript{7}, the $d_{32}$ model was developed simply by assuming sufficiently high turbulence in the reciprocating or pulsed column. Rama Rao\textsuperscript{8} et al. investigated drop size distribution in a reciprocating plate column with plates having smaller hole size and less free area. Smaller $d_{32}$ was found compared with previous studies\textsuperscript{6, 7} in columns with larger plate free area. Columns with small perforation diameter and free areas required less power input but achieved earlier flooding. Later studies\textsuperscript{9-11} were conducted in the Karr column with mass transfer. The drop size distribution and the $d_{32}$ were found also to be influenced by the mass transfer direction. Based on 171 drop size measurements in Karr column without and with mass transfer from literature, Kumar and Hartland\textsuperscript{12} proposed a unified correlation for $d_{32}$. The model consisted of two terms including both low and high agitation. More recently, Stella and Mensforth\textsuperscript{13, 14} compared the existing correlations for drop sizes in Karr columns. The refitted Kumar and Hartland’s model was found to provide the best predictions. They then applied this model to scale up the column to larger diameters. In summary, most studies characterize the drop sizes in the Karr column with an averaged value, the Sauter diameter $d_{32}$. However in reality, the drop size distributions are not uniform along the column. The use of the mean drop diameter cannot adequately describe the distribution profile. In addition, most of the correlations were developed based on turbulent theory derived for the stirred tank, without detailed description of drop breakage and coalescence.
With respect to this, the population balance model (PBM) is a promising approach. It takes into account convective, diffusive and productive terms, including drop breakage and coalescence. Once solving it, the model will predict the drop size distribution along the column. The population balance model has been applied in various types of columns in a stagewise or a differential form, for example, the rotating disc column\textsuperscript{15, 16}, pulsed sieve plate column\textsuperscript{17-20}, Kühni column\textsuperscript{21, 22}, and some newly designed columns\textsuperscript{23, 24}. In these studies, terms of drop behaviors usually come from two ways: from previous literature or from the single drop study\textsuperscript{25}. PBM based on the single drop study were found promising in scale-up design of the column\textsuperscript{26} and has a potential to reduce pilot test requirements\textsuperscript{27}. However, one of the challenges is to obtain a set of robust parameters\textsuperscript{28}, as they are always limited to a certain column geometry and specific test systems. So far, only one study\textsuperscript{29} has applied PBM to the Karr column by using the existing terms from literatures, which was far from optimal. Based on our previous single drop study\textsuperscript{30}, it is necessary to carry on and apply the single drop model to PBM for the Karr column.

In this study, drop sizes were observed and measured in a Karr column. PBM has been established for the column, including both drop breakage and coalescence. The parameters of the model were derived by a two-step optimization. $d_{32}$ predicted by the model were first compared with the experimental results under various operating conditions. Then the model results were validated by comparing to the previous column data with different dimensions as well as the existing correlations in the literature.

2. Population balance model (PBM)
The drop population balance model is formulated based on a drop size distribution function \( P(t, H, d) \) which describes the volume fraction of drops with diameter \( d \) in a control volume of the column at level \( H \) and time \( t \). The general differential form is given as:

\[
\frac{\partial}{\partial t} P(t, H, d) = -\frac{\partial}{\partial H} [P(t, H, d)U_d(t, H, d)] + \frac{\partial}{\partial H} \left[ D_d \frac{\partial P(t, H, d)}{\partial H} \right] + P(H, d) \tag{1}
\]

(term 1) (term 2) (term 3) (term 4)

The integration of \( P(t, H, d) \) over the whole range of drop sizes at height \( H \) leads to the dispersed phase holdup:

\[
\phi(t, H) = \int_0^{d_{\text{max}}} P(t, H, d) \delta d \tag{2}
\]

According to Equation (2), the drop volume balance for a certain drop size is governed by the convective transport (term 2), axial dispersion (term 3) and source term (term 4). The drop breakage and coalescence are taken into account in the source term \( P(H, d) \), so that:

\[
P(H, d) = -PB^-(H, d) + PB^+(H, d) - PC^-(H, d) + PC^+(H, d) \tag{3}
\]

where \( PB^-(H, d) \) and \( PB^+(H, d) \) indicate drop “death” or “birth” due to breakage process, respectively. Similarly, \( PC^-(H, d) \) and \( PC^+(H, d) \) indicate drop loss or appear because of drop coalescence with other drops or form by smaller drops coalescence.

For a Karr column as an agitated column, it can be described as a series of completely mixed stages\(^5\). The back-mixing is mainly due to the agitation and in the continuous phase\(^3^1\). Therefore, the equation (1) is simplified by assuming no axial dispersion in the dispersed phase and given in a stagewise form at steady state as:

\[
P_{n+1}(d_i)U_{d,n+1}(d_i, \phi_{n+1}) = P_n(d_i)U_{d,n}(d_i, \phi_n) + H_p(-PB^-_n + PB^+_n - PC^-_n + PC^+_n) \tag{4}
\]
where $P_n(d_i)$ represents volume fraction of drops of size $d_i$ within stage $n$; $U_{d,n}(d_i, \phi_n)$ is the drop moving velocity with respect to the column walls and is the function of drop size and local holdup; $H_p$ is the height of the stage. The details of breakage and coalescence source terms are summarized in the Table 1.

(Table 1. Breakage and coalescence source terms in the population balance model)

**Drop transport**

The movement of drops in the Karr column is usually caused by vertical rising due to buoyance, random motions due to agitation and influences of other drops. In general, in a counter-current Karr column, the drop velocity $U_d$ can be expressed as:

$$ U_d(d) = U_r(d) - U_c(d) \quad (9) $$

where $U_c$ is the continuous phase velocity in the column and can be calculated as:

$$ U_c = \frac{Q_c}{s(1-\phi)} = \frac{V_c}{(1-\phi)} \quad (10) $$

$U_r$ represents the relative velocity (or slip velocity). By taking drop swarm effects into consideration, it is defined as:

$$ U_r(d) = (1 - \phi)^{n_u} U_{cha}(d) \quad (11) $$

where $n_u$ is the swarm exponent and usually to be 1 or 2. $U_{cha}$ is known as the drop characteristic velocity and can be related to single drop terminal velocity $U_t$ as:

$$ U_{cha}(d) = k_v U_t(d) \quad (12) $$

where $k_v$ is the slowing factor due to column internals and agitation. It depends on the type of the column and with a range of $0<k_v<1$. 
The terminal velocity $U_t$ is a function of drop size and physical properties of the chemical system. It can be measured by single drop experiments in the stagnant fluid. There are many correlations available in literature for systems with low, medium and high interfacial tension. In this study, Klee’s model (Eq(13-15) are in c.g.s unit) was selected to calculate the terminal velocity of drops suitable for high interfacial tension system:

$$U_t(d) = 38.3 \rho_c^{-0.45} \Delta \rho^{0.58} \eta_c^{0.11} d^{0.70} \quad d < d_{crit}$$  

$$U_t(d) = 17.6 \rho_c^{-0.55} \Delta \rho^{0.28} \eta_c^{0.10} \sigma^{0.18} \quad d \geq d_{crit}$$

$$d_{crit} = 0.33 \rho_c^{-0.14} \Delta \rho^{-0.43} \eta_c^{0.30} \sigma^{0.24}$$  

Drop breakage

Equations (5 and 6) in Table 1 represents breakage source terms, one describes the disappearance of drops, while the other describes the formation of drops known as the daughter drop distribution. The breakage rate (or frequency) is defined as the fraction of drop breakup per breakup time. In the liquid-liquid extraction column, the breakage rate $g_n(d)$ is always calculated based on drop breakage probability $p(d)$ and mean residence time of a drop $t_b(d)$ in one stage.

$$g_n(d) = \frac{p(d)}{t_b(d)} = p(d) \frac{\nu_b(d)}{t_b}$$

The breakage probability is defined as the percentage of the breakup drops to the total number of drops, and can be determined by the single drop study. Pioneer studies were carried out in stirred tanks assuming turbulent flow. One of the best-known is proposed by Coulaloglou and Tavlarides (C&T for short in the following discussion).

$$p(d) = \exp\left(-\frac{c_1 \sigma}{\rho d^{2/3} d^{5/3}}\right)$$
This equation has been widely used and become the basis for the population balance model of many researchers.

Earlier studies of breakage probability in columns were conducted by Gourdon et al.\(^{17, 19, 36}\). Single drop experiments were performed in small lab-scale breakage devices, where the internals remained the same as in real columns. They proposed a correlation for breakage probability with the critical Weber number (or critical drop diameter), which is the maximum Weber number (or drop diameter) that drops will not break up under certain operational conditions.

\[
p(d) = \exp\left(-\frac{C_2}{W_e d}\right)
\]

(18)

Recently, based on the experimental and simulation study of the single drop breakage in a reciprocating plate column\(^{30}\), we proposed a correlation for the breakage probability as:

\[
\frac{p(d)}{1-p(d)} = C_{p1} \left(\frac{\rho c \gamma^2 d^3}{\sigma} - \frac{W_{e \text{crit}}}{C_{p2}}\right)
\]

(19)

where \(\gamma\) is the shear rate calculated by the simulation results.

The breakage birth term \(PB^+\) characterized the gain of drops from mother drop breakup with diameter \(d_j\). \(\beta(d_i, d_j)\) is the volumetric density distribution of daughter drops with diameter \(d_i\) generated from this process, as proposed by Bahmanyar and Slater\(^{37}\):

\[
\beta(d_i, d_j) = 3n_{dd}(n_{dd} - 1) \left(\frac{d_i}{d_j}\right)^5 [1 - \left(\frac{d_i}{d_j}\right)^3]^{n_{dd}-1}
\]

(20)

where \(n_{dd}\) is the mean number of daughter drops and can be defined by empirical equation\(^{38}\):

\[
n_{dd} = 2 + C_{n1} \left(\frac{d_j}{d_{\text{crit}}} - 1\right)^{C_{n2}}
\]

(21)
where $d_{\text{crit}}$ is the critical drop diameter, also known as the maximum stable drop diameter. 

$C_{n1}$ and $C_{n2}$ are two adjustable parameters based on the column geometry and chemical system used.

**Drop coalescence**

Coalescence source terms Equation 7 and 8 describe the number of drops with diameters $d_j$ and $d_i$ (or $d_q$) coalesce with a certain rate $\omega(d_i, d_j, \phi)$ (or $\omega(d_j, d_q, \phi)$), generally, only binary coalescence is considered. Drop coalescence rate $\omega$ (or frequency) is often described by the product of drop collision frequency ($h$) and coalescence probability (or coalescence efficiency, $\lambda$).

$$
\omega(d_i, d_j, \phi) = h(d_i, d_j, \phi)\lambda(d_i, d_j, \phi)
$$

(22)

According to Coulaloglou and Tavlarides\textsuperscript{35}, the collision frequency $h$ can be analogous to collision between gas molecules and is widely used.

$$
\begin{align*}
    h(d_i, d_j, \phi) &= c_c \varepsilon^{1/3} \left( \frac{1}{1+\phi} \right)^{1/2} \\
    &= \frac{c_1 \varepsilon^{1/3}}{(1+\phi)^{1/2}} \left( d_i^2 + d_j^2 \right)^{1/2} \\
    &= \frac{c_1 \varepsilon^{1/3}}{(1+\phi)^{1/2}} \left( d_i^2 + d_j^2 \right)^{1/2}
\end{align*}
$$

(23)

The coalescence probability model, however, were developed based on different theories. One is called film drainage model first proposed by Coulaloglou and Tavlarides\textsuperscript{35}, assuming that the forces compress the drops must act for enough time until film drains to a critical thickness so that the film will rupture and drop coalescence will take place.

$$
\lambda \propto \exp \left( -\frac{t_{\text{drainage}}}{t_{\text{contact}}} \right) = \exp \left[ \frac{-C_{c2}b_c\rho_c \varepsilon}{\sigma^2(1+\phi)^3} \left( \frac{d_i d_j}{d_i + d_j} \right)^4 \right]
$$

(24)

Another type is the energy model proposed by Sovová\textsuperscript{39}. It is assumed that the interfacial energy has to be exceeded by the collision kinetic energy for two drops coalesce.
\[ \lambda \propto \exp \left( - \frac{E_{\text{interfacial}}}{E_{\text{kinetic}}} \right) = \exp \left[ -\frac{c_{C2}\sigma(1+\phi)^2(d_i^2+d_j^2)}{\varepsilon^{2/3}p_d d_i^2 d_j^2(\frac{d_i^{1/3}+d_j^{1/3}}{2})} \right] \]  

(25)

The damping factor \((1 + \phi)^{-3}\) is included in all these models to account for the effect of the local energy dissipation at high holdup fractions.

**Solution techniques**

Once the population balance model has been set up and the individual source terms been determined, the model can be solved by different methods, including classes method, Monte Carlo, method of moments. The details of these methods were reviewed in literature\(^{26, 37, 39}\).

In this study, the population balance model was given stagewise and solved in MATLAB R2018b. The column was considered as a series of perfectly mixed stages \((n = 1, 2, 3 \ldots 25)\). Drop size was discretized into \(N_i\) \((i = 1, 2, 3 \ldots)\) classes with interval \(\Delta d = 0.5 \text{mm}\).

Information like column geometry, physical properties, operating conditions, model parameters and initial drop size distribution and holdup was required as model input to initiate the calculation from the first stage. To progress the calculation from \(n^{th}\) to \((n + 1)^{th}\) stage, function fsolve in MATLAB was used with Levenberg-Marquardt algorithm and step tolerance \(10^{-12}\) to obtain \(P_{n+1}\), \(U_{n+1}\) and \(\phi_{n+1}\) respectively.

Optimization of model parameters was conducted by using the Global Optimization Toolbox in MATLAB. The basic idea was to find the parameters set that make the model prediction close to the experimental data as much as possible. As is known from literature\(^{40}\), this kind of inverse problem was always ill-conditioned and sensitive to experimental errors, so that required good quality of experimental data and a stable algorithm. Therefore, a distribution smoothing was completed by fitting the measured raw distribution with the normal
distribution before applying it to the model. This helps avoiding instability and errors when solving PBM during the optimization. Due to the complexity of highly nonlinear models with constraints, gradient-based method was selected to solve the problem using function fmincon with carefully examine the initial range of start points.

3. Experimental

The column experiments were performed in a 50mm inner diameter Karr column. The description of the column could be found in the previous paper\textsuperscript{8}. Stainless steel plates were used in the column with plate open area of 53.55\% and plate spacing 0.05 m. The experiments were conducted under continuous phase velocity (\(V_c\)) ranged from 0.0018 m/s to 0.0052 m/s, and dispersed phase velocities (\(V_d\)) ranged from 0.0005 m/s to 0.0057 m/s. The amplitude (\(A\)) was fixed at 0.013 m, and four reciprocating intensities were chosen, namely, \(Af_1 = 0.00689\) m/s, \(Af_2 = 0.01378\) m/s, \(Af_3 = 0.01924\) m/s and \(Af_4 = 0.02704\) m/s.

VivaSol 2046 (hydrocarbon solvent with 201 - 237 °C boiling point range, kindly supplied by Viva Energy Australia) was used as the dispersed phase and tap water was the continuous phase. The two phases were pumped to the column via centrifugal pumps with the aqueous continuous phase going to the top and the organic phase dispersed from the bottom.

\[(\text{Table 2. Physical properties of the test system (25}^\circ\text{C, 1atm)})\]

The drop size distributions were collected both at the initial stage at the bottom of the column and the middle stage (stage 12) of the column via photographic method using Canon G16 camera. The drop size distribution at the initial stage was used as model input while data at the middle stage was used to compare with model predictions. The captured images were analyzed in ImageJ software. At least 300 drops per photo were counted. In addition, short
videos of drop interactions with the plate were also recorded at the middle stage of the column using Basler acA1920-150um USB 3.0 camera.

Following the capture of drop sizes, the dispersed phase holdup measurement was carried out using the drainage technique. All the inlet and outlet valves were then shut simultaneously. After the dispersed phase was settled, holdup was obtained by measuring the volume causing the interface change.

4. Results and discussion

4.1 Source term model comparison

Breakage model

Drop breakage is important in liquid-liquid extraction as it reduces drop sizes and provides larger surface area. In most of the agitated columns, drop breakage is the dominant factor influencing drop size distribution. This is why some studies made assumptions that neglect drop coalescence and only consider drop breakage in the population balance model.

The selection of breakage model is strongly dependent on the column type. Mechanism of drop breakup varies in different agitated devices. For example, drop breakage in the Kühni column mainly occurs in the vicinity of the impeller or when passing the hole, while for the Rotating Disk Contactor (RDC), the drops were usually breakup by shear stress near the rotor disc. For the Karr column, two types of breakup were observed based on our previous single drop study in Figure 1 (a) & (b), similarly, these two mechanisms were also observed in the column (c) & (d). The first type was breakup due to plate wettability, as shown in (a) & (c). This happened when drops collided with the plate and attached to it, so that the drop moved with the plate up and down and finally detached from it and broke up. The second type of
breakup was due to the shear stress, as shown in (b) & (d). As the drop approached the plate, the shear stress became significant, which caused the drop deformed and elongated. When the shear stress became larger than drop restorative stresses, the drop broke up without wetting of the plate.

(Figure 1. Images of drop breakage in the single drop study and in the Karr column)

To successfully apply PBM to the Karr column, an accurate drop breakage probability model is needed. In the following discussion, three models were compared: the widely-used C&T\(^{35}\) model, Gourdon\(^{36}\) model based on single drop study in the pulsed column, and model based on our previous single drop study for the reciprocating plate column\(^{30}\).

One of the important steps to apply C&T\(^{35}\) and Gourdon\(^{36}\) models was to estimate the energy dissipation rate \(\varepsilon\), as shown in the Equation 17 and 18. According to Hafez et al.\(^{46}\), the energy dissipation rate in the Karr column could be calculated by the following correlation:

\[
\varepsilon = \frac{2\pi^2}{3} \left( \frac{1-e^2}{H_p C_0^2 e^2} \right) (Af)^3
\]

where \(e\) is the open area of the plate, \(H_p\) is the stage height, \(C_0\) is a constant, \(Af\) is the reciprocating amplitude multiplies frequency defined as the reciprocating intensity.

\(C_1\) and \(C_2\) are the constants in the models. They were kept the same values as in the original papers for comparison. Since the C&T\(^{35}\) model has been applied to the Karr column in previous literature\(^{29}\), and \(C_1\) was refitted to be 0.68, which was also included in the discussion.

In order to apply the breakage model (Equation 19) based on our previous single drop study, an estimate of the shear rate \(\dot{\gamma}\) in the column is needed. Based on the CFD simulation results
in the previous work\textsuperscript{30}, the shear rate can be correlated to reciprocating intensity and the plate geometry as the following form, where \(d_h\) is the pitch distance:

\[
\dot{\gamma} \propto k \frac{Af}{ed_h}
\]  \hspace{1cm} (27)

(Figure 2. Estimation of shear rate in the column based on the data from single drop study)

Shear rate is shown to be proportional to \(Af / (ed_h)\) with a slope of \(k=25.855 (R^2=0.9971)\), as shown in the Figure 2. Therefore, once the operational condition and plate geometry is selected, the shear rate in the column can be estimated. Constants \(P_{p1}\) and \(P_{p2}\) were obtained from the single drop study.

Figure 3 (a) compared the three breakage probability models: C&T\textsuperscript{35} model, Gourdon\textsuperscript{36} model and model based previous single drop study\textsuperscript{30}. All models showed the same trend: as the drop diameter increased, drops were more likely to breakup. Larger drops tended to interact with the plate while the smaller drops could easily pass through the holes and stayed intact. Predictions of breakage probability by the model based on our single drop study was higher than other models. It should be noted that the holes diameter on the plate was 12.7 mm in this study, which indicated that drops larger than 12.7 mm should near 100% breakup probability. Both C&T\textsuperscript{35} model and Gourdon\textsuperscript{36} model were not as accurate as the single drop model. One of the reasons was because these models were developed based on different types of agitation with different drop breakup mechanisms. Although the C&T\textsuperscript{35} model was refitted and has applied to the Karr column\textsuperscript{29}, it relied on breakage through intensive turbulence and breakup mechanism under low agitation was not included in the model.

(Figure 3. Drop breakage probability and coalescence probability model comparison)

**Coalescence model**
In most of the agitated columns, coalescence of drops usually occurs at high holdup. Drops will coalesce either due to the random collisions or the smaller drops are over taken by the larger drops while rising in the same direction. Coalescence processes are considered to be more complicated than breakage, more steps are required to accomplish the process and it is extremely sensitive to mass transfer, surfactant and contamination. Therefore, it is difficult to measure the coalescence rate directly via experiments.

C&T model (Equation 24) is the most widely used coalescence probability model in literature. As shown in Figure 3, the coalescence probability decreased as the drop diameter increased. However, the Sovova model (Equation 25) presents a contradictive trend, the coalescence probability increased with an increase in the drop diameter. The total coalescence rate was a function of coalescence probability and collision frequency, both of which were complex functions of various variables. In addition, coalescence and breakage processes occurred simultaneously in the real column, which made it difficult to study coalescence separately. This explained why the influence of the drop diameter on the coalescence probability was presented differently in literature. Therefore, both of the coalescence probability models were used for the population balance model.

4.2 Parameter optimization

To apply the breakage and coalescence model to the PBM, an optimum set of parameters is needed. A broad variety of parameter sets can be found in literature, some of the parameters are dependent on the column geometry while others, like parameter for collision frequency model, has a physical meaning. All of these factors made it challenging to get a set of parameters for a new type of column with different chemical systems and operational conditions. In this study, parameters were classified into three categories based on the
original models: parameters related to drop velocity, breakage and coalescence, summarized in the Table 3.

(Table 3. Parameters in the population balance model)

**Parameter constraints**

Before optimizing the parameters, it is possible to limit the values of some of the parameters as follows. The velocity of drops in the column was significantly influenced by the internals and drop swarm effects, and the latter is also related to the dispersed phase holdup. The slowing factor \( k_v \) characterized the influences of the internals on reducing the velocity of the droplet, thus was dependent on the column geometry\(^{25}\). \( k_v \) was in the range of 0 to 1, when \( k_v = 1 \), there was no slowing effects. The swarm exponent \( n_u \) indicated the extent of the swarm influence. According to Gayler\(^{41}\), the dispersed phase holdup can be related to the phase velocities by mean characteristic velocity as:

\[
\frac{V_d}{\phi} + \frac{V_c}{1-\phi} = U_{cha}(1 - \phi)^{n_u} \tag{28}
\]

The dispersed phase holdup was measured under different operational conditions in the Karr column. Then \( V_d + V_c\phi/(1 - \phi) \) was plotted against \( \phi/(1 - \phi) \), and linear relationship was observed as shown in Figure 4. Therefore, the exponent \( n_u \) was set to be 1.

(Figure 4. Determination of the swarm exponent \( n_u \) by plotting phase velocities \( V_d + V_c\phi/(1 - \phi) \) versus dispersed phase holdup \( \phi/(1 - \phi) \) under different reciprocating intensities)

Breakage related parameters consisted of parameters in the breakage probability model as well as in daughter drops model. As mentioned previously, the breakage probability model
and its parameters were determined based on the single drop study, with $C_{p1} = 0.07$, $C_{p2} = 1.28$. Two adjustable parameters in the number of daughter drops model were dependent on the column type and test system, some existing values could be found in literature\textsuperscript{38}. Parameters for the coalescence model were more complicated. $C_{c2}$ in collision frequency model had physical meaning related to critical film thickness. For example, $C_{c2}$ in C&T\textsuperscript{35} model was defined as:

$$C_{c2} = \frac{1}{h_c^2} - \frac{1}{h_0^2}$$  \hspace{1cm} (29)

where $h_c$ is the critical film thickness and $h_0$ is the initial film thickness. Therefore, the corresponding lower and upper bounds for the parameters can be calculated based on the thickness ranges in literature\textsuperscript{50}. In summary, there are five parameters need to be determined in this study: $k_v$, $C_{n1}$, $C_{n2}$, $C_{c1}$ and $C_{c2}$.

Two-step optimization

To optimize the five parameters, the objective function can be written as:

$$\min \sum_{i=1}^{N_i} [P_{mid,cal}(d_i) - P_{mid,exp}(d_i)]^2$$  \hspace{1cm} (30)

where $P_{mid}$ is the drop size distribution at the middle stage (stage 12), $N_i$ is the total number of drop discretized classes. The goal was to find the parameters set that make the model prediction close to the experimental data as much as possible. Optimization was conducted using a two-step method.

In the first step, velocity and breakage model parameters, $k_v$, $C_{n1}$ and $C_{n2}$ were optimized under the low holdup and high reciprocating frequency conditions with only drop breakage was considered in PBM. Experimental data under $Af_3$ and $Af_4$ with the lowest dispersed
phase velocity \( (V_d = 0.0022 \text{ m/s}) \) was used as database. Parameters \( k_v \), \( C_n1 \) and \( C_n2 \) were solved and then averaged. The second step was to optimize the parameters \( C_{c1} \) and \( C_{c2} \) in the coalescence models. For this case, experimental data under \( A_f_3 \) and \( A_f_4 \) with the highest dispersed phase velocity \( (V_d = 0.0057 \text{ m/s}) \) was used as database and breakage and coalescence terms were both considered.

Different parameters values of \( C_{c1} \) and \( C_{c2} \) can be found in literature and summarized in Table 4. Especially for \( C_{c2} \) in C&T\textsuperscript{35} model, some studies gave the value in the range of \( 10^{11} \) to \( 10^{13} \), while others obtained the parameter to be in the range \( 10^6 \) to \( 10^8 \), which were in quite different magnitude.

(Table 4. Parameters for C&T\textsuperscript{35} and Sovová\textsuperscript{39} coalescence probability model in literature)

Therefore, in order to solve for the second-step successfully using the gradient-based optimization method, the starting points should be chosen carefully. For C&T\textsuperscript{35} coalescence probability model, a coarse data mapping for \( C_{c2} \) was done for both ranges: \( (10^6,10^8) \) and \( (10^{11},10^{13}) \) and then compared. Smaller value of the objective function was found to be 0.000444 as \( C_{c2} \) in the range of \( (10^6,10^8) \), compared to the value 0.00333 as \( C_{c2} \) in the range of \( (10^{11},10^{13}) \). Therefore, the initial range for \( C_{c2} \) was set in \( (10^6,10^8) \). Figure 5 (a) shows the variation of the objective function with \( C_{c1} \) and \( C_{c2} \) for C&T\textsuperscript{35} model in data mapping. Both parameters had influence on the objective function. Compared with the data mapping for Sovová\textsuperscript{39} model in a range of \( (10^{11},10^{13}) \) for \( C_{c2} \) in Figure 5 (b), \( C_{c2} \) did not have much effect on the objective function as \( C_{c1} \). After knowing the coarse information of the objective function, the optimization problem was solved accurately with the above initial parameter ranges and the final set of five parameters were summarized in Table 5.
4.3 Influence of operating conditions

After obtaining the optimized parameters, Sauter diameter $d_{32}$ can be plotted along the column. Figure 6 shows the change in the drop size distribution for one condition $V_c=0.0036$ m/s, $V_d=0.0036$ m/s and $A_f=0.02704$ m/s. As drops entered the column, repeated breakage and coalescence occurred and change the drop size distribution. It can be found that drop breakage was dominant and resulted in a narrower distribution in the middle of the column.

Influence of the different operating conditions on the drop size distribution was shown in Figure 7. Figure 7 (a) compares the cumulative drop size distribution under four different reciprocating intensities ranging from 0.00689 m/s to 0.02704 m/s, with phase velocities $V_c=0.0036$ m/s and $V_d=0.0057$ m/s. Model predictions (lines) are well correlated with the experimental data (symbols). Two coalescence models: C&T$^{35}$ model and Sovová$^{39}$ model were plotted for comparison. Both models predicted well without much difference, which was in agreement with the findings in Kühni column by Jildeh$^{50}$. Especially under the high reciprocating intensity, $A_f=0.02704$ m/s, the predictions by the two models were almost the same. This was because at higher reciprocating intensities, the process of drop breakage was more dominant than drop coalescence. Although the two models predicted the coalescence probability in a contradictory trend, the value only ranged between 0 to 1. By multiplying with the collision frequency as well as the number of drops, little difference could be found
between using these two models.

With the decrease of $Af$, larger drop sizes were observed. However, deviations were found between the model predictions and the experimental data for the lowest intensity, $Af = 0.00689$ m/s. It might because the coalescence models used in the PBM were based on previous literature, which were not developed for the Karr column at low agitation. In terms of drop breakage, drops were more likely to breakup due to plate wettability at low $Af$, which was not covered in the single drop breakage model. Therefore, future studies are needed to develop a reliable single drop coalescence model and take into account the plate wettability effects on drop breakage model for the Karr column.

(Figure 7. Influence of operating conditions on drop size distribution in the Karr column)

Figure 7 (b) compared three continuous phase velocities $V_c=0.0018$ m/s, 0.0036 m/s and 0.0052 m/s, and two dispersed phase velocities $V_d=0.0036$ m/s and 0.0022 m/s, all of these experiments were run with the reciprocating frequency $Af=0.01924$ m/s. Experimental data shows no significant changes in the drop sizes among different phase velocities, which was in agreement with previous studies conducted in the Karr column$^6, 13$. Model predicted well with the experimental data, as the phase velocity only occurs in the velocity term and was nearly a magnitude smaller than the drop velocity, thus had little influence on the results.

4.4 Comparison with previous studies

As shown in Figure 8, the Sauter diameter ($d_{32}$) in the column was calculated based on PBM predictions and compared with the existing experimental data (Figure 8(a)) and correlations (Figure 8(b)) in literature. Table 6 summarized the experimental data from four studies in the Karr columns with various geometries and test systems.
However, all of these studies only focused on the Sauter diameter and have not measured the initial drop size distribution. Therefore, to apply PBM to these studies, the initial drop size and the distribution were estimated by the following equations\textsuperscript{55-57} with the parameters suggested by Korchinsky\textsuperscript{58}:

\begin{align}
\label{eq:31}
d_{in} &= 3.8\left(\frac{\sigma}{\Delta \rho g}\right)^{1/2} \\
\label{eq:32}
d_m &= d_{in}[1 + \eta \exp\left(\frac{1}{4\delta^2}\right)] \\
\label{eq:33}
q_0 &= \frac{\delta}{\sqrt{\pi}} \frac{d_m}{d(d_m-d)} \exp\left[-(\delta \ln\frac{\eta d}{d_m-d})^2\right]
\end{align}

Figure 8(a) compared the PBM predictions with the experimental data in literature. Good agreement was achieved as most of the data were within the $\pm 30\%$ range. Baird's\textsuperscript{6} data were predicted well as the original study was conducted in the Karr column of a very similar diameter to in our study (5.00cm compared to 5.08cm). In addition, the test system was similar, kerosene-water in Baird's\textsuperscript{6} study and VivaSol 2046 (mainly kerosene) in our study. Therefore, good agreement was found between this comparison. Although Aravamudan et al.\textsuperscript{9} conducted their experiments in the same column as Baird\textsuperscript{6}, the system was switched to a high interfacial tension system (50mPa $\cdot$ s), where the data were not perfectly predicted.

Half of the data in the study of Bensalem\textsuperscript{10} fell outside the range, as they carried out the experiments in a column with different geometries as well as different test systems. Given the experimental errors and their influence on the estimation of the initial drop sizes, the predictions considered to be reasonable. Data based on the study of Mensforth\textsuperscript{13} were well
predicted although in a different test system, the experiments were completed in the same column as in this study.

(Figure 8. $d_{32}$ predictions compared with previous studies)

Figure 8 (b) compared the measured Sauter diameter $d_{32}$ in this study with predictions by PBM and other correlations from literature. All the correlations presented here used the parameters based on the original study. Predictions by PBM, Baird$^6$ and Kumar$^{12}$ were in good agreement with the measured drop sizes, since all the models have considered low agitation (or drop coalescence) and high agitation (or drop breakage). Correlations by Boyadzhiev$^7$ and Rama Rao$^8$ gave higher predictions of $d_{32}$. This was because these correlations were derived based on the fully turbulent theory and failed to predict in a wide range of operating conditions. Joseph$^{11}$ and Bensalem$^{10}$ developed their correlations based on the database consisted of mass transfer, which involved interfacial instabilities like Marangoni effects that would change the drop breakage and coalescence behaviours and resulted in differences in the drop size distribution.

Conclusions

A stagewise population balance model (PBM) was developed for the Karr column. Drop breakage mechanisms were the same as in the single drop study. Drop breakage probability model based on single drop study was found to be the best to use.

A two-step optimization method was used to estimate model parameters. Optimization starting points were determined by data mapping to ensure global optimization results. Good agreement was achieved by comparing the model predictions with the experimental data.
Drop sizes decreased as the reciprocating intensity increased, but not affected much by the phase velocities.

The PBM framework was then applied to the Karr columns with different systems and geometries. Model predicted well with the previous experimental data. Finally, the PBM was compared with some existing correlations. Models only based on turbulent theory cannot predict well in a wide range, others consisted of low agitation term can cover more general cases.

**Notations**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Reciprocating amplitude, m</td>
</tr>
<tr>
<td>$Af$</td>
<td>Reciprocating intensity, m·s$^{-1}$</td>
</tr>
<tr>
<td>$C_0, C_1, C_2, C_{p1}, C_{p2}, C_{n1}$</td>
<td>Model parameters</td>
</tr>
<tr>
<td>$C_{n2}, C_{c1}, C_{c2}$</td>
<td></td>
</tr>
<tr>
<td>$d$</td>
<td>Drop diameter, m</td>
</tr>
<tr>
<td>$d_{32}$</td>
<td>Sauter mean diameter, m</td>
</tr>
<tr>
<td>$d_{cr it}$</td>
<td>Critical drop diameter, m</td>
</tr>
<tr>
<td>$d_h$</td>
<td>Pitch distance of the plate, m</td>
</tr>
<tr>
<td>$d_{in}$</td>
<td>Inlet mean drop size, m</td>
</tr>
<tr>
<td>$d_m$</td>
<td>Maximum diameter, Mugele-Evans function, m</td>
</tr>
<tr>
<td>$D_d$</td>
<td>Axial dispersion coefficient, m$^2$/s</td>
</tr>
<tr>
<td>$e$</td>
<td>Plate open area</td>
</tr>
<tr>
<td>$f$</td>
<td>Reciprocating frequency, Hz</td>
</tr>
<tr>
<td>$g$</td>
<td>Breakage frequency, s$^{-1}$</td>
</tr>
<tr>
<td>$h$</td>
<td>Collision frequency, m$^3$/s</td>
</tr>
<tr>
<td>$h_0$</td>
<td>Initial film thickness, m</td>
</tr>
</tbody>
</table>
Critical film thickness, m

Column height level, m

Effective height, m

Plate spacing, m

Slowing factor

Plate thickness, m

Number of daughter drops

Number of drops with diameter $d_i$

Swarm exponent

Number of plates

Total number of drop size intervals

Drop breakage probability

Source term in the population balance model, $1/(m \cdot s)$

Volumetric distribution of drops at n stage

Initial volumetric drop size distribution in dispersed phase at initial stage, $1/m$, $P_n = \phi_n q_n$

Volumetric drop size distribution at the middle stage, $1/m$

Continuous phase flow rate, $m^3/s$

Reynolds number

Sectional area of the column, $m^2$

Time, s

Breakage time, s

Continuous phase velocity, $m/s$

Characteristic velocity, $m/s$
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$U_d$</td>
<td>Drop rising velocity, m/s</td>
</tr>
<tr>
<td>$U_r$</td>
<td>Relative velocity, m/s</td>
</tr>
<tr>
<td>$U_t$</td>
<td>Terminal velocity, m/s</td>
</tr>
<tr>
<td>$v$</td>
<td>Volume of drop, m$^3$</td>
</tr>
<tr>
<td>$V_c$</td>
<td>Calibrated continuous phase velocity in the experiments, m/s</td>
</tr>
<tr>
<td>$V_e$</td>
<td>Effective volume, m$^3$</td>
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<tr>
<td>$We_{crit}$</td>
<td>Critical Weber number</td>
</tr>
<tr>
<td>$We_d$</td>
<td>Drop Weber number</td>
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<tr>
<td>$We_m$</td>
<td>Modified Weber number</td>
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</table>

**Greek letters**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\beta$</td>
<td>Daughter drop size distribution</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Energy dissipation rate, m$^2\cdot$s$^{-3}$</td>
</tr>
<tr>
<td>$\mu_c, \mu_d$</td>
<td>Viscosity of the continuous phase and dispersed phase, mPa·s</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Coalescence probability</td>
</tr>
<tr>
<td>$\rho_c, \rho_d$</td>
<td>Density of the continuous phase and dispersed phase, g/cm$^3$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Interfacial tension, mN/m</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Coalescence rate, m$^3$/s</td>
</tr>
<tr>
<td>$\dot{\gamma}$</td>
<td>Shear rate, s$^{-1}$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Dispersed phase holdup</td>
</tr>
<tr>
<td>$\eta, \delta$</td>
<td>Distribution parameter, Mugele-Evans function</td>
</tr>
</tbody>
</table>

**Subscripts**

<table>
<thead>
<tr>
<th>Subscript</th>
<th>Description</th>
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<tbody>
<tr>
<td>$c$</td>
<td>Continuous phase</td>
</tr>
<tr>
<td>$d$</td>
<td>Dispersed phase</td>
</tr>
<tr>
<td>$i, j, q$</td>
<td>Drops with different diameters</td>
</tr>
<tr>
<td>$n$</td>
<td>Stage number</td>
</tr>
<tr>
<td>$mid$</td>
<td>Middle stage of the column</td>
</tr>
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</table>
Acknowledgement

The work is performed under the joint-PhD program between Tsinghua University and the University of Melbourne. Postgraduate research support from both universities is gratefully acknowledged. The authors also thank Viva Energy (Australia) for providing VivaSol 2046 as solvent for the column work.

Data availability statement

Drop Sauter diameter data analyzed in Figure 8(a) are openly available at:

Baird et al.: http://dx.doi.org/10.1016/0009-2509(77)80029-8, reference [6].

Aravamudan et al.: https://doi.org/10.1021/ie980605s, reference [9].

Bensalem et al.: https://doi.org/10.1016/0255-2701(86)85017-6, reference [10].

Mensforth: Library of University of Melbourne (Accession number: melb.b3096928), reference [13]. Request is needed for access: https://library.unimelb.edu.au/collections/special-collections/rare-books/australiana/umtheses. For help, please contact the corresponding authors.

References


Figure 1. Images of drop breakage in the single drop study and in the Karr column. 
((a), (b) Images by high-speed camera Photron FASTCAM Mini WX50, $A_f=0.0202$ m/s, plate open area=54.4%, Drop diameter = 3.24 mm, kerosene-water system; (c), (d) Images by Basler acA1920-150um USB 3.0 camera at stage 12 in the Karr column, $A_f=0.0312$ m/s, plate open area=53.55%, $V_c=0.0018$ m/s, $V_d=0.0005$ m/s, VivaSol-water system)
Figure 2. Estimation of shear rate in the column based on the data from single drop study

(Circles: data from single drop study[^30]; Line: linear approximation)
Figure 3. Drop breakage probability and coalescence probability model comparison
((a) Breakage probability versus drop diameter with single drop breakage model\textsuperscript{30}, Gourdon\textsuperscript{36} model, original and refitted C&T\textsuperscript{35} model; (b) Coalescence probability versus drop diameter with C&T\textsuperscript{35} model and Sovová\textsuperscript{39} model. Both plots with $V_c=0.0036$ m/s and $V_d=0.0018$ m/s, $Af=0.016$ m/s)
Figure 4. Determination of the swarm exponent $n_u$ by plotting phase velocities $V_d + V_c\phi/(1 - \phi)$ versus dispersed phase holdup $\phi/(1 - \phi)$ under different reciprocating intensities.
Figure 5. Data mapping to find possible initial range for start point for the second-step optimization with (a) C&T$^{35}$ and (b) Sovová$^{39}$ coalescence probability model
Figure 6. Change of Sauter diameter $d_{32}$ along the column ($V_c=0.0036\text{m/s}$, $V_d=0.0036\text{m/s}$ and $Af=0.02704\text{ m/s}$)
Figure 7. Influence of operating conditions on drop size distribution in the Karr column (a) influence of reciprocating intensity $Af$ with $V_c=0.0036$ m/s and $V_d=0.0057$ m/s; (b) influence of phase velocities $V_c$ and $V_d$ with $Af = 0.01924$ m/s (Solid lines represent C&T$^{35}$ model prediction, dash lines represent Sovová$^{39}$ model prediction)
Figure 8. $d_{32}$ predictions compared with previous studies. (a) $d_{32}$ predictions by the PBM compared with experimental data in literature$^{6,9,10,13}$; (b) Experimental measured $d_{32}$ in this study compared with the predictions by the PBM and the correlations in literature$^{6-8,10-12}$. 
Table 1. Breakage and coalescence source terms in the population balance model

<table>
<thead>
<tr>
<th>Source term</th>
<th>Equations</th>
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<tr>
<td>Breakage death term</td>
<td>$PB^- = P_n(d_i)g_n(d_i)$</td>
</tr>
<tr>
<td>Breakage birth term</td>
<td>$PB^+ = \sum_{j&gt;i} P_n(d_j)g_n(d_j)\beta(d_i, d_j)\Delta d_j$</td>
</tr>
<tr>
<td>Coalescence death term</td>
<td>$PC^- = P_n(d_i) \sum_{d_{\text{min}}}^{(d_{\text{max}}^2-d_i^2)^{1/3}} \frac{P_n(d_j)}{v_j} \omega_n(d_i, d_j, \phi_n)\Delta d_j$</td>
</tr>
<tr>
<td>Coalescence birth term</td>
<td>$PC^+ = \frac{v_j}{2} \sum_{d_{\text{min}}}^{d_i} \omega_n(d_j, d_q, \phi_n) \frac{P_n(d_j)}{v_j} \frac{P_n(d_q)}{v_q} \left(\frac{d_i}{d_q}\right)^2\Delta d_j$</td>
</tr>
<tr>
<td></td>
<td>($v_l = v_j + v_q$)</td>
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Table 2. Physical properties of the test system (25°C, 1atm)

<table>
<thead>
<tr>
<th></th>
<th>Density $\rho$ kg/m$^3$</th>
<th>Viscosity $\mu$ mPa·s</th>
<th>Interfacial tension $\sigma$ mN/m</th>
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<tr>
<td>Tap water (c)</td>
<td>998.0</td>
<td>1.002</td>
<td>28.85</td>
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<tr>
<td>VivaSol 2046 (d)</td>
<td>780.2</td>
<td>1.656</td>
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Table 3. Parameters in the population balance model

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameters</th>
<th>Description</th>
<th>Equations</th>
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<tr>
<td>Velocity</td>
<td>$k_v$</td>
<td>Slowing factor, related to internals(^{49}), no slowing effect when $k_v = 1$</td>
<td>(12)</td>
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<tr>
<td></td>
<td>$n_u$</td>
<td>Swarm exponent, usually 1 or 2, or as a function of Reynolds number(^{32})</td>
<td>(11)</td>
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<tr>
<td>Breakage</td>
<td>$We_{\text{crit}}$</td>
<td>Critical Weber number</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$C_{p1}$</td>
<td>Parameters in breakage probability model, based on the single drop study</td>
<td>(19)</td>
</tr>
<tr>
<td></td>
<td>$C_{p2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$C_{n1}$</td>
<td>Parameters in number of daughter drops model, dependent on the column type and test system(^{38})</td>
<td>(21)</td>
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<tr>
<td></td>
<td>$C_{n2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coalescence</td>
<td>$C_{c1}$</td>
<td>Parameter in collision frequency model(^ {50})</td>
<td>(23)</td>
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<tr>
<td></td>
<td>$C_{c2}$</td>
<td>Parameter in coalescence probability model, related to the critical film thickness(^ {35})</td>
<td>(24-25)</td>
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</table>
Table 4. Parameters for C&T\textsuperscript{35} and Sovová\textsuperscript{39} coalescence probability model in literature

<table>
<thead>
<tr>
<th>Author</th>
<th>$C_{c1}$</th>
<th>$C_{c2} \ (m^{-2})$</th>
<th>Coalescence probability model</th>
</tr>
</thead>
<tbody>
<tr>
<td>C&amp;T\textsuperscript{35}</td>
<td>2.8×10\textsuperscript{-6}</td>
<td>1.83×10\textsuperscript{13}</td>
<td>C&amp;T\textsuperscript{35}</td>
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<tr>
<td>Attarakih\textsuperscript{51}</td>
<td>6.94×10\textsuperscript{-2}</td>
<td>1.3×10\textsuperscript{11}</td>
<td>C&amp;T\textsuperscript{35}</td>
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<tr>
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<td>C&amp;T\textsuperscript{35}</td>
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<tr>
<td>Ribeiro\textsuperscript{52}</td>
<td>1×10\textsuperscript{-3}</td>
<td>5.446×10\textsuperscript{6}</td>
<td>C&amp;T\textsuperscript{35}</td>
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<tr>
<td>Kabouche\textsuperscript{53}</td>
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<td>2.289×10\textsuperscript{7}</td>
<td>C&amp;T\textsuperscript{35}</td>
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<td>Simon\textsuperscript{54}</td>
<td>1.8×10\textsuperscript{-2}</td>
<td>9.25×10\textsuperscript{-1}</td>
<td>Sovová\textsuperscript{39}</td>
</tr>
<tr>
<td>Jildeh\textsuperscript{50}</td>
<td>3.05×10\textsuperscript{-2}</td>
<td>8.99×10\textsuperscript{-1}</td>
<td>Sovová\textsuperscript{39}</td>
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<tr>
<td>Jildeh\textsuperscript{50}</td>
<td>6.21×10\textsuperscript{-3}</td>
<td>4.05×10\textsuperscript{-3}</td>
<td>Sovová\textsuperscript{39}</td>
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Table 5. Final set of the five parameters for the population balance model

<table>
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<tr>
<th>$k_v$</th>
<th>$C_{n1}$</th>
<th>$C_{n2}$</th>
<th>$C_{c1}$</th>
<th>$C_{c2}$</th>
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<tbody>
<tr>
<td>0.8708</td>
<td>0.0100</td>
<td>4.1476</td>
<td>With C&amp;T\textsuperscript{35} model</td>
<td>0.0075</td>
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<td>2.9521×10\textsuperscript{7}</td>
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<td></td>
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<td>With Sovová\textsuperscript{39} model</td>
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<td></td>
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<td></td>
<td>0.0029</td>
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<td>Research</td>
<td>System</td>
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<td>Plate open area</td>
<td></td>
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<tr>
<td>---------------------</td>
<td>----------------------</td>
<td>-----------------</td>
<td>-----------------</td>
<td></td>
</tr>
<tr>
<td>Baird(^6), 1973</td>
<td>Kerosene(c)-water(d)</td>
<td>5.08cm</td>
<td>0.55</td>
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<tr>
<td>Bensalem(^10), 1986</td>
<td>Toluene(d)-water(c)</td>
<td>7.60cm</td>
<td>0.60</td>
<td></td>
</tr>
<tr>
<td>Aravamudan(^9), 1999</td>
<td>Isopar M(d)-water(c)</td>
<td>5.08cm</td>
<td>0.56</td>
<td></td>
</tr>
<tr>
<td>Mensforth(^13), 2006</td>
<td>Kerosene + 10% TBP(c)-water(d)</td>
<td>5.00cm</td>
<td>0.452</td>
<td></td>
</tr>
</tbody>
</table>
Author/s:
Zhang, J; Li, W; Mumford, K; Fei, W; Stevens, G; Wang, Y

Title:
Drop sizes and population balance model for a Karr column

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