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PROBABILITY and PATERNITY:
The Utility of Probability Theory in
the Legal Determination of Facts in
Issue with particular reference to
the resolution of paternity disputes.

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This thesis is, except where due acknowledgement
has been made in the text, my work alone. It
does not include material for which any other
University degree or diploma has been awarded.
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The airline industry (meaning not just the airlines themselves but also the airframe manufacturers, engine and component manufacturers and various Government organizations) would have you believe that airline travel is less risky than climbing into bed. Some days it is, some days it isn't. On May 25, 1979, it was 100 percent fatal for the 271 on board [American Airlines flight 191 departing from Chicago]. The day before, on the same flight, it was 100 percent safe. Statistics can devil the hell out of you if you let them, but you pay your money and you take your chances, and in this game, undelivered goods are nonreturnable.


In this thesis it is proposed to examine the forensic application of probability to judicial or curial decision making, both in general and with particular reference to the determination of parentage such as is required in a paternity suit.

Chapter 1 provides some well known examples of fallacies derived from an imprecise understanding of probability including some previous attempts to introduce conclusions based on the mathematical probability theory into legal reasoning. Because a number of writers have postulated a non-mathematical probability it is necessary to distinguish between the various meanings of the term and to discuss the non-mathematical concepts. The chapter concludes with the introduction of "transitivity", a concept necessary to allow comparison between or to rank, mathematically or
otherwise, conclusions based on the knowledge available to a decision maker.

The mathematical basis of Probability theory is covered in chapter 2 including the arithmetical rules used to obtain a combined mathematical probability value of a number of incidents occurring together when each incident has its own individual probability value. Further, a graphical representation or Venn Diagram is introduced to display probability values.

Decision making based on available or known probability values of the occurrence of incidents is dealt with in chapter 3 with a discussion of a measure of the reliability of such decision making.

Chapter 4 serves as an introduction to chapter 5 and sets out the evidentiary requirements of proof necessary to establish facts in issue in a legal hearing including the necessity or at least desirability of corroboration of some forms of evidence. The chapter also includes a discussion on quantifying the evidentiary value of corroborating evidence. Chapter 5 considers the evidentiary requirements in cases involving disputed parentage and necessarily includes a description of genetic principles with particular reference to the detection of genetic markers in human tissue and the conclusions which can be drawn from the detected presence of such markers.

The concluding discussion is presented in chapter 6.
1. INTRODUCTION

"Your prompt decision to attack," said General Grant on a certain occasion to General Gordon Granger, "was admirable; you had but five minutes to make up your mind in."

"Yes, Sir," answered the victorious subordinate, "it is a great thing to know exactly what to do in an emergency. When in doubt whether to attack or retreat I never hesitate a moment - I toss up a copper."

"Do you mean to say that's what you did this time?"

"Yes, General; but for Heaven's sake don't reprimand me: I disobeyed the coin."

Ambrose Bierce, "Indecision", The Devil's Dictionary (1911).

Only 10% of accidents are caused by those who are intoxicated, the other 90% are caused by sober drivers. The obvious answer is to get the sober ones off the road and leave it safe for the rest of us.

Dr. Gerald Milner (Director of Alcoholic and Drug Dependent Services), Melbourne Symposium on Drug Abuse, 19 May 1976.

I abhor averages. I like the individual case. A man may have six meals one day and none the next, making an average of three meals per day, but that is not a good way to live.


Given the recent interest displayed in the application of Probability Theory and Statistical Methods to the legal finding of fact - one recent text 1 contains a five page bibliography of approximately 150 entries of which more than half are of date 1970 or later - it is thought desirable
to scrutinize the elements of the basic theory upon which some of the more sophisticated concepts are founded. In this area fallacies and misconceptions abound and they are not exclusive to the unversed layman - Leibniz thought it just as easy to throw a twelve with a pair of dice as to throw an eleven, d'Alembert could not see that the results of tossing a coin three times are the same as tossing three coins at once, and he believed (as many 'amateur' gamblers persist in believing) that after a long run of heads, a tail is more likely. Similarly to Leibniz, d'Alembert could not understand why two heads (or for that matter, two tails) will occur less frequently than one head and one tail when two coins are tossed and it would appear that this erroneous interpretation of both Leibniz and d'Alembert is shared by some post-graduate law students whose minds have been specifically directed to probability theory. Sir Richard Eggleston confesses to an early error in combination assessments and no less an authority than Weaver has fallen into the error of both d'Alembert and Leibniz when their respective chestnuts are reclothed as the problem of the three chests and Martin Gardner's second ace and Huff's three card game. Cohen's error in enumerating as eight the number of integers greater than ten and less than twenty is not a misconception of probability, it is a mere arithmetical blunder. It is not difficult to appreciate how it could be written that "Charles Sanders Pierce once observed that in no other branch of mathematics is it so easy for experts to blunder as in probability theory." Eggleston has suggested that
it may be necessary to get rid of juries and indeed most members of the legal profession and to recruit more numerate people to do the fact-finding for us\textsuperscript{14}. One can only assume that Eggleston has in mind people more numerate than the college mathematics instructor who gave expert testimony in the notorious \textit{Collins} case\textsuperscript{15}. How is the layman to question the prophets if the prophets themselves experience difficulty in interpreting their own faith\textsuperscript{16}? Such a question must be asked given the recent resurgence of writings both for and against\textsuperscript{17} the application of probability theory to the determination of facts in the context of our legal system. Such fact-finding is entrusted to a jury or, in some cases, a judge sitting alone with no qualifications required regarding the fact-finder's knowledge or learning related to probability theory. Of historical note only it is suggested that the resurgence of writings described above was triggered by the notorious \textit{Collins} case\textsuperscript{18}. That it is a resurgence follows from the fact that some of the earliest writings on probability theory were directed at legal fact-finding\textsuperscript{19}.

Notwithstanding the advocacy of utilizing probability theory for legal fact-finding the courts have declined to take up the challenge with few exceptions. Finkelstein writes that when

\begin{quote}
\textit{[c]onfronted with statistical evidence of uncertain validity, judges tend to avoid the un congenial and exposed task of evaluation in favour of intuitive appraisals that cannot be as readily attacked or a simple refusal to make any finding}.\textsuperscript{20}
\end{quote}

Finkelstein notes that statistical and probabilistic
evidence has been accepted in cases involving blood typing\textsuperscript{21} and jury discrimination cases\textsuperscript{22}. Whereas in Sargent v. Massachusetts Accident Co \textsuperscript{23} and Collins\textsuperscript{24} (the first being a civil case and the second being criminal) evidence based on mathematical probability alone was suggested as insufficient to discharge a burden of proof\textsuperscript{25}, although in Collins, at first instance, such evidence was admitted and it was only on appeal that the admissibility of such 'pseudostatistics'\textsuperscript{26} was rejected. In 1984 the Full Court of the Supreme Court of South Australia, while accepting evidence that most, but not all, drivers with a blood alcohol level of 0.15 would be incapable of exercising effective control of a motor vehicle, held that the evidence was not probative with regard to the particular defendant in the case before the court.\textsuperscript{27} In the case, which was a civil suit and not a criminal trial, the Chief Justice expressly rejected the argument based on mathematical probability alone\textsuperscript{28} and the other two members of the court did so by implication. The Chief Justice expressly adopted the reasoning of Williams in that the evidence should focus on the defendant\textsuperscript{29} and consequently rejected Eggleston's opinion\textsuperscript{30} in that the Chief Justice was clearly of the opinion that the statistical fact that a particular proposition is true of the majority of persons cannot of itself amount to legal proof on the balance of probabilities that the proposition is true of any given individual\textsuperscript{31}. In that case the court was being asked to consider the distinction between a unique and unknown past event and a body of evidence founded on many known past events\textsuperscript{32}. 
The distinction was discussed by Emery J in the following terms:

That in one throw of dice there is a quantitative probability, or greater chance, that a less number of spots than six will fall uppermost is no evidence whatever that in a given throw such was the actual result. Without something more, the actual result of the throw would still be utterly unknown. The slightest real evidence that sixes did in fact fall uppermost would outweigh all the probability otherwise.

In those cases noted by Finkelstein where statistical and probabilistic evidence was held admissible such evidence was admitted as an adjunct to other probative evidence.

Nomenclature

The answer is yes or no depending on the definition.


Professor John Cohen of Manchester University suggests two kinds of probability - mathematical and psychological. Kahneman and Tversky use the term "subjective" probability to describe the latter as does Hamburg. More recently Kahnemann and Tversky have used the term "Bayesian" for such intuitive assessments of probability. Hodges and Lehmann use "personal" probability. L. Jonathan Cohen of Oxford University postulates two kinds of probability - Pascalian for the classical mathematical theory of probability and Baconian for what Cohen chooses to describe as "inductive" probability. These adjectives were chosen by Cohen in recognition of Pascal's contribution to the classical mathematical theory.
of probability and Bacon's position in history as the first to properly emphasize inductive methods as the basis of scientific procedure respectively.\textsuperscript{42} Notwithstanding his honouring of Bacon, Cohen alone claims sole credit for the formulation of his non-Pascalian Baconian inductive probability.\textsuperscript{43} By 1980 when Cohen had apparently vanquished the Pascalian, he trained his guns on the Bayesians. However, a close reading of Cohen's more recent writings reveal that he has merely renamed as "Bayesian" his 1977 mathematical Pascalian probability. It is unclear whether Cohen decided on the new name as a more pejorative term than Pascalian or to demonstrate his recent acquaintance with Bayes.\textsuperscript{44} Given that it was Bayes who first provided the mathematical basis to Bacon's inductive logic\textsuperscript{45} it would appear a strange choice to describe that probability theory opposed to Cohen's own inductive Baconianism. To further confuse the matter, Kahneman and Tversky have used the term Bayesian to describe the non-mathematical subjective probability.\textsuperscript{46}

Given the usual meaning of the words 'deduction' and 'induction' as used in mathematics, logic and philosophy it is somewhat odd that Cohen has elected to describe his non-mathematical probability as 'inductive'.\textsuperscript{47} It is inductive logic that permits a general conclusion to be made on the basis of a limited number of observations. Much of what is accomplished with mathematical probability is in fact inductive such as production run sampling\textsuperscript{48} or opinion polling\textsuperscript{49} where conclusions are drawn regarding a much larger parent population on the basis of the results.
obtained from a smaller sample. Induction permits a tongue-in-cheek journalist to predict that the mile will be run in three minutes and 45 seconds in 1987 and in three minutes and 30 seconds in 2023 and that on August 1, 2528 the mile will finally be run in no time at all - a feat which will thereby ruin athletics as a spectator sport\textsuperscript{50}.

Ayer describes three different kinds of judgment of probability\textsuperscript{51} - the first relating to the mathematical calculus of chances with an example given of the probability of throwing double six with a true pair of dice is one in 36. The second is statistical judgment such that there is a slightly better than even chance that any given unborn infant will be a boy. Ayer's third is a credibility judgment such as an assessment of the chances that Britain will join the Common market. This latter probability is equivalent to the non-mathematical probabilities previously described whether they be personal, psychological, subjective, inductive et cetera. The first two are both mathematical, the first being derived from the Principle of Indifference\textsuperscript{52} or inductive reasoning from known possibilities and the second from observed statistical frequencies or deductive reasoning from known prior occurrences. For present purposes it is only necessary to distinguish between the mathematical and the non-mathematical probabilities.

Henceforth the above adjectival labels will not be used except where necessary and the term 'probability' alone will be used to refer to the classical mathematical theory of probability.
Cohen's Baconianism

Aristotle could have avoided the mistake of thinking that women have fewer teeth than men by the simple device of asking Mrs Aristotle to open her mouth.

Attributed to Bertrand Russell.

Given Sir Richard Eggleston's discussion of Cohen's inductive probability it is not proposed to traverse ground already covered. However two general observations unrelated to Cohen's theory can be made regarding his appreciation of the theory of probability and an argument by analogy used by him to support the validity of his theory.

(i) When, as on page 18 and again on page 19 of his The Probable and the Provable, Cohen asserts that the probability of a number's being prime, if greater than 10 and less than 20 may be said - informally - to be .5, because out of eight such numbers just four are primes, he is in error. Of the nine such numbers (11, 12, 13, 14, 15, 16, 17, 18 and 19) just four (11, 13, 17 and 19) are primes. Such an assertion "is like the thirteenth stroke of a crazy clock, which not only is itself discredited but casts a shade of doubt over all previous assertions." And, it might be said, over any future assertions. The explanation may well lie with that suggested by Mr. Cohen himself, that the text of his book available in Melbourne, Australia differs from the one published in Oxford. Or perhaps the basic rules of arithmetic and number
theory undergo a transformation on the journey between the two cities.

(ii) In support of his proposition that his theory is valid he provides an analogy with geometrical space. Because there exists non-Euclidean geometry besides Euclidean geometry, it is just as incorrect to suppose that the only theory of probability is a Pascalian one. Leaving aside the meaningless reference in the index of his book to non-Euclidean geometry, Cohen raises this analogy on three occasions. The mere fact that Cohen makes the point with his analogy three times does not necessarily make the analogy true notwithstanding Lewis Carroll. His geometrical analogy fails when one considers that Euclidean geometry is but a special instance of the generalized non-Euclidean geometry. Nowhere does Cohen claim that probability is a special instance of his generalized Baconian theory.

It is worth considering the exchange between Professor Williams and Cohen. In reviewing the book Williams concluded that Cohen has, of course, demonstrated that legal proof cannot be accurately expressed in mathematical terms, but it hardly needed a book to convince us of that. For his trouble Williams is classified as a Pascalian and guilty of misplaced mathematicisation. At that point Williams gracefully retired from the battle writing "It is difficult to avoid the feeling that Mr. Cohen is immune
against criticism because he is determined never to admit an error."

The Property of Transitivity

It is a mark of immaturity to expect the same degree of precision in human affairs as in mathematics.

Aristotle (attributed)

Transitivity is a binary relation such that if it holds between A and B and between B and C, it must also hold between A and C. If Arthur is taller (older, shorter) than Betty; and Betty is taller (older, shorter) than Charles then it follows that Arthur is taller (older, shorter) than Charles. The relationship between the members of the group under consideration is said to be transitive.

Not all such relationships are transitive: that Essendon is a better football team and consequently defeats Fitzroy by 15 points in a game and Fitzroy is a better team than Geelong and wins a game between these two teams by 25 points does not necessarily entail that when Essendon plays Geelong, Essendon will win the game or that Essendon will win such a match by 40 points. Such intransitivity does not faze the sports follower; it can be attributed to the weather on the day, the composition of the team, the ground conditions and many other factors — indeed, there would be little spectator interest in such games if they were subject to transitivity and the results a foregone conclusion.
Unlike sports matches from which influencing factors cannot always be kept constant games of chance and the assessment of such chances do exhibit transitivity. Thus, in the long run, there is a higher probability of throwing a head in a toss of a coin ($\frac{1}{2}$) than there is of drawing at random a spade card from a properly shuffled deck of cards ($\frac{1}{4}$). And there is a higher probability of drawing such a spade card than there is of throwing a three using a true die ($\frac{1}{6}$). This necessarily implies that there is a higher probability of throwing a head in a toss of a coin than there is of throwing a three using a true die.

But this is not always so.

Efron has designed a set of four dice in which the first is more likely to exhibit a higher score than the second with odds two to one in favour of the first (or with probability $\frac{2}{3}$). Similarly the second is more likely (with odds of two to one in favour) to score higher than the third and the third die is more likely (again with odds of two to one in favour) to score higher than the fourth. Yet, when it comes to considering the first and fourth dice, the fourth die is more likely, with odds of two to one in favour, to score higher than the first. The four dice are shown below in 'unfolded' form to display their six faces.
An example of this non transitive property from real life was found in the 1974 world heavyweight boxing title held in Zaire between Ali and Foreman. Foreman had recently beaten another boxer (Frazier) and that boxer had previously beaten Ali. The conclusion could be that Foreman should beat Ali, a conclusion supported by the odds being offered against Ali of 4:1. However, ranking the three boxers on the skills of speed, power and style according to press reports the ranking displays a non transitive property. Thus Ali outranked Foreman in two of the three skills, a result borne out by the result of the actual match.

<table>
<thead>
<tr>
<th>Speed</th>
<th>Power</th>
<th>Style</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ali</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Frazier</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Foreman</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

Of a similar nature to Efron's non-transitive dice is the reversal of preference depending on the field of choice. It was first noted by Condorcet in the context of preferential voting and has been since "rediscovered" by Lewis Carroll, Black and Blyth. Where there is a choice between candidates A and B, the electorate will vote in A - similarly the electorate prefers A to C and B to C. Yet in an election where all three A, B and C
stand, C obtains the greatest number of votes with A gaining the least. With preferential voting, as in Australia, the loser's votes are redistributed and the electoral contest is reduced to a two candidate run-off between B and C of which B is successful.

Consider the cumulative data from two sources. This is known as Simpson's Paradox although it is much older than his 1951 paper. If a choice of two hats (black and grey) which contain respectively five red and six white chips while the second hat contains three red and four white chips then choosing the black hat will provide the greatest likelihood of selecting a red chip (5/11 compared to 3/7). Similarly a second set of black and grey hats, the first containing six red and three white and the other containing nine red and five white the choice of the black hat again provides the greatest likelihood of selecting a red chip (6/9 compared to 9/14). However if the contents of the two black hats are combined as well as combining the contents of the two grey hats, the grey hat will provide the greatest likelihood of selecting a red chip: 12/21 compared to 11/20. That this is not an arithmetical novelty from the desk of a statistician is demonstrated by its occurrence in real life.

The purpose of these three examples is merely to demonstrate that the application of probability theory may provide different results from those intuitively expected and also the limitations of probability models.
based upon over-simplifying the real problem so as to make it tractable to probability models. Problems of the real world can only rarely be shown to have a field of possibilities which are disjunctive, exhaustive, exclusive and equiprobable\textsuperscript{72}. None of these examples are true paradoxes in that they are contradictory - at most they may be said to be paradoxical in that they are not in accord with the results expected intuitively\textsuperscript{73}. 
Notes to Chapter 1: Introduction


7. Ibid. 161.


9. Ibid. 90-2; Eggleston op. cit. 28 and 250.


16. *Supra notes 5-10.*

17. *Supra note 1,* the majority of authors advocate the use of probability theory.


23. (1940) 307 Mass. 246, 29 NE (2nd.) 825.


25. 307 Mass 246, 250-1, 29 NE (2nd.) 825, 827 per Lummus J.


28. *Ibid.* 419 and 33 per King CJ.


34. Supra notes 21-22.

35. Finkelstein, op. cit. 20-1, 43, 54-5.


46. Supra note 39.


49. Hamburg M., op. cit. 249 ff.; Hodges J.L. and Lehmann E.L., op. cit. 231-2; Reichman W.J., op. cit. 279-84.
50. Kitson T., "The Ultimate Mile", National Times, August 10, 1984; see also Mark Twain, Life on the Mississippi (1881) ch. 17.


52. See ch. 2.1 infra.


57. Ibid. 95.


67. Kotz and Stroup op. cit. 146.


73. Kotz and Stroup op. cit. 148.
2. MATHEMATICAL THEORY OF PROBABILITY

It may seem to your reporter to be 'in defiance of astronomical odds", that the same outlet should sell the two highest payoff tickets in New York State lottery history. But this event is really less improbable than it appears.

Since there are 2,100 such outlets in the state, it is easily shown mathematically that the odds against this occurrence would be a maximum of 2,099 to 1 if the outlets had equal ticket sales, and lower if unequal, as is actually the case.

By way of comparison, this event is much more likely to happen than drawing four-of-a-kind in a randomly dealt poker hand.


Dr. Williams said of AIDS that it seemed if sensible precautions were taken 'you have got as much chance of picking it up as being kicked to death by a duck'.


There are available two methods of assessing numerical values of probability\(^1\). One is based on pure mathematics\(^2\) and the other on observed data. It is expected that the two methods give similar results but they may not do so. Thus a long series of observations of a tossed coin is expected to provide a frequency statistical probability value of approximately one-half for the probability of throwing a head. The a priori judgment of probability based on the pure mathematical calculus of chance gives this value of one-half. Yet the frequency statistical judgment may not give the expected value of one-half. For example, of 1,000 tosses, heads may be observed on 410
occasions and of 10,000 such tosses the same coin may return 6,500 heads.

There are a limited number of event sets that permit assessments by both methods. The frequency distribution of blood groups in the ABO system for example is A 42%, B 8%, O 47% and AB 3%. Unless the assessor of such values is given cause the best distribution that the pure mathematician could provide is A 25%, B 25%, O 25% and AB 25%.

Similarly the only realistic assessment of customer's colour preference for Ford passenger vehicles is one based on observation of frequency. The estimate that white is favoured by 65% of purchasers could only be made by the pure mathematician with only 1.54 colours available. And it follows that such an estimate is related to the number of colours available and should the manufacturer increase (or decrease) the range of colours available then the simplistic pure mathematical assessment must be reassessed.

It seems clear in an event set of colours that not all colours are of equal weight of probability. However a more subtle case is that of the sex of new born infants. There does not appear to be any biological or other reason why there should be a preponderance of male births over female or vice versa. Thus a usual value of one-half is assigned to each possible event. And yet long term
frequency observations disclose a small but distinct bias favouring a male birth: ~ 0.513 and correspondingly 0.487 for a female birth. Although the variation from 0.5:0.5 seems small it is extremely significant given the event set well in excess of 20,000,000 births per year. Although there is not an agreed reason or cause it would appear to be a property of the father who can be the only source of the Y chromosome necessary for a male birth. Huff notes the speculation following study of large families involving only male children being an unknown genetic property of the father.

2.1 The Principle of Indifference

When I was a Judge at first instance, sitting alone, I could and did do justice. But when I went to the Court of Appeal of three, I found that the chances of doing justice were two to one against.


Classical probability theory arose from an interest in games of chance. The very simplicity of these games led to the adoption of several basic assumptions. It is suspected that because these assumptions possess such credibility they are now accepted without question. Although the assumptions may be described as axiomatic to basic probability theory, it is feared that they may not safely be extended to some of the more complex problems which are thought by some to be amenable to probability theory.
The first basic assumption has been described as "the primitive theory"\textsuperscript{12} wherein the probability of an event is the number of outcomes corresponding to the event as a ratio to the total number of possible outcomes. An example is the probability of drawing a white ball from a bag containing a number of white, black, yellow, red and blue balls. The drawing of any black, yellow, red or blue ball is equated with a failure or the drawing of a non-white ball - thus there are only two possible outcomes of such a drawing, either a white ball is drawn or a non-white ball is drawn. Thus the primitive probability value associated with drawing a white ball is 1 to 2 or 0.5.

The primitive theory was supplemented with an account of equal likelihood\textsuperscript{13} leading to the Principle of Indifference as it was named by Keynes\textsuperscript{14}, it having previously been known as the Principle of Insufficient Reason. It may be set out as follows with respect to a fair or unbiased coin. Given that the coin has only two faces with no possibility of it falling on its edge there is no apparent reason why one face should be more favoured than the other. Thus, in the absence of any evidence to the contrary, the probability associated with either face cannot exceed that associated with the other face. Because the total of the probabilities must be a certainty\textsuperscript{15}, the probability associated with each face is one-half of a certainty. Assigning a numerical value of one to a certainty, it follows that the probability of a head is one-half and similarly for a tail. This reasoning may be applied to
a fair die leading to an assigned probability value of one-sixth for each of the six faces. Similarly, a standard well shuffled pack of 52 cards gives a probability value that any particular card will be chosen at random of $1/52$.

The Principle can now be restated generally such that, if there are no grounds for believing that any one of $n$ mutually exclusive outcomes is more likely to occur than any other, a probability of $1/n$ may be assigned to each outcome.

Shannon has formulated an elegant concept of probability that is dependent on maximising a quantity defined by him as the entropy or uncertainty subject to imposed constraints. Based on theoretical considerations of the comparison between transmitted and received messages in communication channels, Shannon "invented" Information Theory. Central to his thesis is the function "average information content" which is of a form and substance closely analogous to the thermodynamic quality entropy. Shannon adopted this term for use in his Information Theory. It is, as in thermodynamics, a measure of uncertainty. Thermodynamic entropy is a measure of the randomness, degree of disorder, or chaos of a system. The recognition that entropy may be interpreted as missing information is attributed to Boltzmann. The uncertainty is that uncertainty about the outcome. If the event takes place the uncertainty is removed. That is, information has been
transmitted and received and the amount of information received is equal to the reduction in uncertainty.

By maximizing the entropy any bias towards a particular outcome is minimized. That is, maximizing the entropy subject to the constraints imposed by the available information gives the minimum bias associated with these constraints. Where there are $n$ possible outcomes, each outcome having a probability of occurrence $P_1, P_2, \ldots, P_n$ the entropy is a function of these (unknown) probabilities of occurrence as independent variables and maximizing the entropy function by partial differentiation with respect to each independent variable gives $n$ separate partial differential equations which can be solved by the method of Lagrange (undetermined) multipliers yielding values of $P_1, P_2, \ldots, P_n$ for maximum entropy. Thus a six-faced die subject only to the constraints that the sum of the probabilities $P_1 + P_2 + P_3 + P_4 + P_5 + P_6$ must equal one (a certainty) and that the average value (or expectation) of each face is 3.5 will yield, upon maximizing the entropy function

$$P_1 = P_2 = P_3 = P_4 = P_5 = P_6 = \frac{1}{6}$$

The concept can be further illustrated by considering two schemes: scheme A and scheme B. Scheme A is such that there are two possible outcomes (1 and 2), each outcome with probability one-half while Scheme B is such that of
its two possible outcomes (3 and 4), 3 has probability 0.99 and 4 has probability 0.01. The uncertainty associated with scheme A is greater than that of scheme B. Scheme A is less biased towards one of its component outcomes than is scheme B. The scheme of minimum bias possesses the greatest uncertainty.

There is no detriment in using empirical evidence to provide an assessment of assigned probability values: over a long series of tosses of a fair coin the expectation is that an approximately equal number of heads and tails will result and thus a probability value of one-half can be assigned to each. This technique is known as frequency or Bernoulli trials and for the purpose of assigning probability values to the faces of a coin or die or to the cards of a deck, there is no real difference to the end result except that it is doubted anyone has had the patience to actually conduct a large number of such trials. If someone does carry out such trials that person is generating statistical data. Were someone foolish enough to do so, the empirical data may turn out to be of little value. Consider the following example from A.J. Ayers: given that there is a 5/36th chance of throwing an 8 when two dice are thrown (these odds are slightly more than or longer than 6 to 1 against) someone has brought himself to doubt that the odds against throwing an 8 are more than 6 to 1 and decides to test the question by experiment. After recording the results of many thousands of throws he finds that the proportion of times in which his pair
of dice has yielded an 8 is as high as one in five:

What has he proved? Perhaps no more than that his dice are biased; at most that tossing dice is not an affair of chance in the way it has been taken to be, but certainly nothing that has any bearing on the theory of probability.

Leaving aside classical seventeenth century probability theory games of chance, consider now a more complicated arrangement. A biased die, for example, a die that is asymmetrically weighted or perhaps of a matchbox shape where of the six faces there are three sets of two similar faces per set, each set differing from the others. It is suggested that a priori deduction is valueless in these circumstances and the only meaningful estimate of probability is that gained from a frequency trial or statistic. Certainly the Principle of Indifference is inappropriate because it is no longer possible to assert that there is no apparent reason why one face should be more favoured than another. Although such a die possesses six possible outcomes it is not true that

Similarly the probability of any one outcome in a six-outcome game of chance is one-sixth, because a rule which says

From an outcome's being either A, B, C, D, E, or F infer its being A has a one-sixth reliability wherever it applies.

Thus the Principle of Indifference must be carefully qualified and applied to avoid pitfalls. There is no shortage of examples to illustrate these; it being possible to "prove" the existence of life on Mars, that wagering on the existence of God is a better proposition than wagering against and that the hair of the inhabitants
of Saturn must be either one of two colours. Chapter 4 of Keynes's "Treatise" provides further examples of the absurdities arising from incautious application of the Principle. Ayers provides the following problem attributed to Watling:

Suppose we are following a man along a road and reach a place where the road divides into three, two paths climbing the hillside, one lying in the valley. Knowing nothing but that the man, now out of sight, will take one of three paths, how are we to estimate the probability that he will take the path lying in the valley? If we follow the classical procedure of assigning equal probability to equal possibilities, and if we regard it as equally possible that the man will take any one of the three paths, we shall have to conclude that the chance of his taking the valley path is one in three. But we might regard it as equally possible that he will go into the valley or into the hills, and in that case it would follow that the chance of his taking the valley path was one in two. These conclusions are mutually incompatible, but in default of further information there is nothing to choose between them. Another problem, that will not be discussed here is the presence (or absence) of random choice by the traveller. Can we, in truth, assert that the traveller will choose which path to follow randomly?26

Another illustration: suppose there is a cube in the next room whose size has been selected by a randomizing device. The cube's edge is longer than one metre and less than three metres. How does one estimate the probability that the cube's edge is between one and two metres as compared with the probability that it is between two and three metres? Unlike the first, this illustration is defined to incorporate randomness. Without additional information is it not reasonable to invoke the Principle of Indifference and take each probability as one-half? It is not. If the cube's edge ranges between one and two metres its volume ranges between one and eight cubic metres.
But in the range of possible edges between two and three metres the volume ranges between eight and twenty-seven cubic metres - a range approximately three times that of the volume range associated with an edge length between one and two metres. If the Principle of Indifference is applicable to the two ranges of edge lengths, it is violated by the equivalent ranges of volumes. The imprecision of defining how the cube's "size" was randomized and the very ambiguity of "size" gives no clue to guide what is no more than a guess. Any application of the Principle is meaningless.\(^7\)

The two illustrations given above are hypothetical. There is no strong objection to such examples devised within an institute as an educational tool provided that such examples are not permitted to escape and terrorize the general public. It is feared that this indifferent beast has escaped.

In the discussion following a seminar conducted by Mr. L.J. Cohen at the Monash University Law School on September 4th, 1980, Sir Richard Eggleston noted that a present justice of the Australian High Court was, while practising at the bar, engaged in a case involving a motor accident where all the occupants of a military vehicle were killed and there was no evidence at the accident scene to indicate which of the deceased occupants was the driver of the vehicle. The present justice formed the opinion that the probability of any one of the deceased being the
driver was equally likely, i.e., given (say) eight deceased occupants then the probability that any one was the driver would be one-eighth. Sir Richard further suggested that the Justice's opinion has not changed since that particular case.  

A possibly similar case went to the Victorian Court of Criminal Appeal in 1975 where the appeal on two grounds was dismissed: The Queen v. Stephenson. It cannot be stated firmly that Stephenson was a case involving probability theory in that one of the grounds for appeal was the refusal of the trial judge to allow certain evidence to be introduced in the trial. It is mere conjecture that the defendant was attempting to put an argument using probability theory and the argument was founded on the Principle of Indifference.  

The facts of the case were that the appellant Stephenson was convicted on three counts of culpable driving and one of causing grievous bodily harm. These charges arose from the deaths of three of the four occupants of a Fiat car which was making a right-hand turn which was involved in a collision with the appellant's oncoming vehicle. The testimony of the sole surviving occupant, a Miss B, was inconclusive as to the identity of the driver of the Fiat but did suggest that a Mr. S was probably the driver in that he had been driving the car a short time prior to the accident. The three deceased had been tested for blood-alcohol levels, the results of which differed widely
and in the case of Mr. S gave no support to any suggestion that his driving ability had been impaired by the consumption of alcohol. The appellant sought to have introduced into evidence the results of any such test performed on Miss B and it was the refusal by the trial judge to allow this evidence that formed the basis of one of the grounds for appeal.

It is at this point that conjecture is required in lieu of the appellant's inchoate argument. It is suggested that the appellant was attempting to show that the Fiat's driver was affected by alcohol. If this were the case the appellant need only show the possibility that the accident was caused by the driving of the Fiat to forestall the prosecution discharging the burden of the criminal standard of proof in order to secure his acquittal in the trial proper or a reversal of his conviction on appeal. The appellant was not bound by the criminal standard of proof nor was he bound by the civil standard – he only needed to raise a reasonable doubt that the accident was caused by himself. To do so the appellant desired to attribute to the Fiat's driver possible impairment of driving capability caused by the consumption of alcohol. Not being able to show that the driver was in fact affected by alcohol, it not being known who was the driver, the appellant sought to show that of the four occupants, three were adversely affected by alcohol. By applying the Principle of Indifference it would follow that there was a one in four chance that Mr. S, whose ability had not
been so impaired, was the driver and a three in four chance that one of the other occupants was the driver and was adversely affected by alcohol. It was not strictly necessary for the appellant to show this in that, if it is assumed that Miss B was not affected by alcohol, there remained an even chance that the driver was or was not so affected. It is important to note that not only was the appellant not required to "prove" his interpretation of the facts beyond a reasonable doubt (the criminal standard of proof borne by the prosecution), he was not required to do so on the balance of probabilities (the civil standard of proof) either. For his purposes the introduction of a reasonable doubt would have been sufficient and thus, the showing that there was an even probability of the Fiat's driver being affected was all that or even more than the appellant required.

As previously indicated, the above reconstruction is conjecture only with regard to an argument that may have been advanced by the appellant had the trial judge permitted him to do so. As such it is an example of the application of the Principle of Indifference and in its construction it is similar to the case of the military vehicle related by Sir Richard Eggleston. One difference however, is that in the Stephenson case there was evidence, admittedly inconclusive, adduced to show that a short time before the accident occurred, the then driver of the Fiat was not affected by alcohol such that his driving ability was impaired\textsuperscript{32}. While this evidence was inconclusive it
is sufficient to negate the essential requirement for the application of the Principle: that there is no reason why one of the possible outcomes should be more likely than the other possible outcomes.

Consider now the *Stephenson* case where the evidence suggesting that Mr. S was the driver of the Fiat has not been adduced. The *Stephenson* case is now similar to the bare case involving the military vehicle outlined by Sir Richard Eggleston. Is it now possible to apply the Principle of Indifference and draw a conclusion that there was a three-quarters (or a one-half - depending on the result of a test that was not conducted on Miss B) probability that the Fiat was driven by a person whose capability was affected by alcohol? That there was (say) a one-eighth probability to be allocated to each of the eight deceased soldiers in the military vehicle? It is submitted that it is not so possible. As with the hypothetical case of Watling's traveller and the three paths, the element of randomness is missing. Furthermore the requirement that all outcomes are equally likely follows from a lack of information suggesting otherwise. The principle was after all that of insufficient reason prior to its renaming by Keynes as that of indifference. In the four examples so far considered there is certainly a lack of information but it is necessary to distinguish between cases involving such a lack of information. It is one thing to seek in vain a clue suggesting bias in a coin or die and another to abstain from seeking such relevant information.
Probability theory is not to be used in lieu of hard evidence or as an excuse for failing to seek out such hard evidence. L.J. Halstead makes the same point thus:

It must be realised that a probability statement about a particular situation depends on the state of our knowledge concerning that situation.... Without investigation our answer to the question - What is the probability that Christmas day in the year 2000 is a Sunday? - would be 1/7 but we could resolve our doubt by reference to an appropriate calendar and so give an 0 or 1. (In fact the day is Monday and the answer is 0.)

Thus it is submitted that the party desiring to invoke the Principle of Indifference is bound to provide sufficient evidence justifying its use, i.e., evidence that there is a lack of information.

Who owned the Fiat? In the case of the army vehicle, who was authorized to drive it?
Who possessed a driver's licence?
Who in fact could drive?
Who usually drove the vehicle?
Did any of the occupants of the Fiat have a propensity to drive while affected by alcohol? Conversely, did any have a propensity to decline to drive when so affected?
How old was each occupant and did any possess physical disabilities precluding them from driving?

It is suggested that, in general, informative answers to such questions will tend to negate the applicability of the Principle of Indifference. The answers favouring the validity of the Principle's use would be similar to
the following:

It cannot be determined who owned the vehicle, or None of the occupants owned the vehicle (it having been stolen or borrowed by the occupants as a group), or The vehicle was jointly owned by all the occupants. Either all or none possessed a driver's licence. Either all or none could in fact drive. Either all or none usually drove the vehicle.

All the occupants had a propensity to drive while drinking - this is sufficiently uninformative to allow the invocation of the Principle while evidence of a propensity to refuse to combine drinking and driving would imply that the unaffected occupant was the driver.

Each occupant was of an age and physical capacity to drive. Evidence that one of the occupants was an infant quadraplegic would support the inference that it was highly probable that this occupant was neither the driver nor affected by alcohol. Unless this type of information is sought and is found to be unavailable, the Principle of Indifference should not be resorted to.

There is no judicial enunciation regarding the above suggestion that the party seeking to rely on the Principle is bound to justify its use. Indeed, it has been suggested that at least one member of the present High Court of Australia leans the other way. Eggleston's discussion of T.N.T. Management v. Brooks suggests that during argument before the High Court, counsel for the appellant asserted that
there were only three possible conclusions to be drawn from the evidence and that each of these three possible conclusions was equiprobable - that the plaintiff's husband was negligent, that the defendant's employee driver was negligent, and that both the plaintiff's husband and the defendant's employee driver were negligent. Murphy J then pointed out that if that were the case, the appellant must lose the appeal. Does the above suggestion offend against the requirement of our legal system that the judge's role is that of an umpire leaving the adversaries to present their case as they see fit and the arbiter of fact being restricted to the evidence adduced by the parties? It is submitted that the above suggestion does not so offend. The party seeking to rely on the Principle is suggesting to the arbiter that a particular inference may (or should) be drawn from the adduced evidence. That party should show cause why one inference is to be drawn at the expense of other feasible inferences.

Thus it is concluded that, except in the most simple cases, the Principle of Indifference cannot be assumed and must be justified. Further it is suggested that applying the Principle to people is a much more difficult and complex proposition than that presented by a coin or die. While it may be true that all men are created equal, it must be recognized that the equality may have, and most probably has, been lost by the time such men have travelled sufficiently far through this world for their affairs to have come to the attention of a court. It may be that
Keynes did us a disservice in renaming the Principle of Insufficient Reason: the "indifference" in the Principle relates to the Principle itself, not to the standard of care to be used when applying the Principle.

Broad distinguished between real life occurrences and simplified probability models thus:

Probabilities are only measurable in the comparatively rare case where we have a field of possibilities which can be split up disjunctively into exhaustive, exclusive, and equiprobable alternatives. This does happen in games of chance and in the "bag" problems in which mathematicians exercise themselves, but not in many other cases.

2.2 The Law of Large Numbers

Krogh said contemptuously, 'Why the quickest way? This is the last gamble some of us will have. We may as well enjoy it. I say a coin.'

'It won't work,' the clerk said, 'You can't get an even chance with a coin.'


Ayer distinguished between judgments of a *priori* probability (or the mathematical calculus of chance based upon the Principle of Indifference) and judgments of frequency distribution as a measure of probability. There are two different approaches to the latter judgment of probability values, both of which are unsatisfactory because they both involve consideration of an infinity.
The first is the limiting frequency whereby an infinite series of trials are conducted and the event under consideration occurs \( m \) times in the first \( n \) trials. Consequently probability is defined to be the limiting value of \( m/n \) as \( n \) tends to infinity. Unfortunately there is not (and there cannot be) proof that such a limit exists. The second approach involves an infinite number of trials with the subset of trials in which the event under consideration occurs compared to the set of all trials.

The unsatisfactory nature of the two approaches cannot be eliminated but can perhaps be minimized by considering a large number of trials only. Thus, in one sense the practical resolution of these two approaches tends toward the practical resolution of the judgment based on the Principle of Indifference although from an opposite direction. With the latter, it is necessary to postulate an increasing number of trials to give meaning to the probability value assessment while the frequency distribution approaches require a reduced number of trials to eliminate the intractable problems associated with the infinite number.

That there is a probability of one-half for throwing a head (and also a one-half for a tail) in flipping a coin does not mean that one single toss of the coin will see it teetering between heads and tails. Thus, while it appears axiomatic that as the number of "trials" is increased the totality of the results of those trials will
more accurately reflect the assessment of probability value. In fact Bernoulli postulated and provided a proof of his "Golden Theorem" : the law of large numbers.

The probability tends towards certainty that the proportion of successes observed in a binomial experiment approaches without limit the true probability of success on a single trial as the number of trials approach infinity.

Bernoulli was able to prove, although modern proofs are easier than his, that

\[ \frac{m}{n} \approx p \text{ for large } n \text{ or, where } c \text{ represents an arbitrary error limit for which it is required that the difference between } \frac{m}{n} \text{ and } p \text{ be less than } c, \]

\[ P\left(\left|\frac{m}{n} - p\right| > c\right) \leq \frac{1}{4nc^2} \text{ for a binomial distribution.} \]

It should be noted that

(i) the law of large numbers does not "prove" experimental long-run frequencies to be stable; and

(ii) the law does not prove that \( \frac{m}{n} \) lies between \( p \pm c \) where \( c \) can be made arbitrarily small; it merely proves that as \( n \) tends to infinity, the probability that \( \frac{m}{n} \) lies between \( p \pm c \) tends to 1 (a certainty).

While the law does not prove experimental long-run frequencies to be stable, the real world is not under the sway of mathematical arguments; mathematical models are realistic only to the extent that their conclusions correspond to observation. Long-run stability is an empirical fact. The law of large numbers merely asserts that the probability model being used is sufficiently realistic to agree with this fact.
Thus one of the most striking things about probability theory is that it tends to explain the (perhaps otherwise) strange fact that events which are individually capricious and unpredictable are capable of displaying a very stable average performance when such individual events are considered en masse. Schroedinger named this property of individual events the "order from disorder" principle:

The disintegration of a single radioactive atom is observable (it emits a projectile which causes a visible scintillation on a fluorescent screen). But if you are given a single atom, its probable lifetime is much less certain than that of a healthy sparrow. Indeed, nothing more can be said about it than this: as long as it lives (and that may be for thousands of years) the chances of its blowing up within the next second, whether large or small, remain the same. This patent lack of individual determination nevertheless results in the exponential law of decay of a large number of radioactive atoms of the same kind.

Schroedinger's principle is in accord with Shannon's entropy wherein maximum randomness leads to statistical regularity.

Notwithstanding the apparently paradoxical principle, there can be hidden pitfalls such as the gambler's fallacy that a long run of red at the roulette table increases the probability that the next colour to come up will be the black. Reference has already been made to Ayer's experimentalist who set out to test empirically the odds of making a point of eight and failing to obtain results agreeing with the theoretical probability even after many thousands of throws. Similarly it would be a foolish man who argued that because the total number of motor vehicle accidents in the current year had already risen
to the average of previous years, he could drive as recklessly as he pleased.\textsuperscript{56}

Further, the law of large numbers does not necessarily support the inference that a large sample is unlikely to deviate in character from the parent population unless the parent population is finite and the unexamined or unsampled proportion of the parent population is small.\textsuperscript{57}

The corollary of the law of large number (or long-run stability) of random events is that individual events are not necessarily amenable to probability theory except in the instance where one wishes to assess the probability of a single event (with however an appreciation on the long run). Thus the gambler can assess the odds for a particular event on the basis that in the long run his decisions will be "correct" more often than not. The occurrence of a single event lacks the weight or leverage to affect the long run: the expectant father desiring a son who knows that in previous years the birth ratio has exhibited a 10,000:9,500 bias towards male births and that this year the current score is 7,500:8,000 may have to reconcile himself with the fact that this year the ratio will be 9,999:9,501 or that this year is, for unexplained reasons, an abnormal year.\textsuperscript{58}

Perhaps the simplest illustration of the inherent limitation of the law of large numbers as applied to the single event is a statistic such that the average household
in Australia today has 2.7 children, 1.4 cars and the breadwinner has been married 1.7 times. These qualities can only take integer values and the non-integer statistics merely reflect an overall view of a large number of integer qualities "averaged" out to give such statistics.

While such statistics and even assessments of probability values may be useful to those persons dealing with numerous "units" it is feared that a legal hearing to determine facts is a unique "one-off" problem that does not readily lend itself to analysis based on larger groups: General propositions do not decide concrete cases.

To those dealing with numerous "units", the individual cases are of no concern. So long as the estimated proportion of its clients die in any given year, it does not matter to the insurance company who dies and who lives. Thus the management of a Victorian lottery can state that "lotteries are about luck, not statistics." This is true for the lotteries clientele, but for the lottery management it is the reverse - luck has nothing at all to do with the management's income and outgoings, it is all statistically predictable.

2.3 Arithmetic Rules of Probability

Mathematics, a veritable sorcerer in our computerized society, while assisting the trier of fact in the search for truth, must not cast a spell over him.

Quite apart from anything else, the elementary mathematics of judges are prone to error, except in the case of Lord Denning who was a wrangler, and his maths are too good for anyone else to understand.


The arithmetic rules for considering the combined or cumulative affect of various probabilities is based on set theory and are well established for simple cases. There is continuing discussion and disagreement with regard to the more complex cases involving different probabilities.

Because these rules are based on set theory the algebra of set theory and diagrammatic representations of sets can be used for probability. Diagrammatic representations are now known as Venn Diagrams after the English logician John Venn who first used such representations to illustrate the symbolic logic of sets.

The sample space is a representation of unit area, for example the rectangle depicted in figure 1 can represent a deck of cards, the sides of a coin, the population of a country, the annual output of passenger vehicles from a motor vehicle manufacturer et cetera.

Figure 1: Venn Diagram

Area = 1.00

representing the sample space of events.

Consider now the event space representing a subset of the sample space, for example the event of suit of spades in a deck of cards.
Figure 2: Venn Diagram representing the sample space of events (a deck of cards) and the event representing the suit of spades.

The area of the sample space remains 1.00 and the area of the event representing the suit of spades is 0.25 or one-quarter of the sample space. The area of the event space represents the probability of the event - in this case the probability of a spade is 0.25 representing the thirteen possible spades in a deck of 52 cards. The area of the event space outside (or disjoint from) the event space representing the spade suit is 0.75 representing the 39 possible non-spades in a deck of 52 cards.

Thus $\Pr(\text{spade})$ or $\Pr(S) = 0.25$

and $\Pr(\text{non-spade})$ or $\Pr(\overline{S}) = 0.75$

where the complement of the event "spade" is "non-spade" and is represented by a raised bar. It follows that

$$\Pr(S) + \Pr(\overline{S}) = 1;$$

therefore $\Pr(\overline{S}) = 1 - \Pr(S)^{67}$.

Other event spaces can be represented on the same Venn diagram, for example Figure 3 represents the events spade suit, diamond suit and the sub-event" Ace of spades. The corresponding probabilities represented by the areas are:
Pr(S) = 0.25 and Pr(\overline{S}) = 0.75
Pr(D) = 0.25 and Pr(\overline{D}) = 0.75
Pr(AS) = 1/52 = 0.02 and
Pr(\overline{AS}) = 49/52 = 0.98

Figure 3: Venn Diagram representing the sample space of events (a deck of cards) and the events of
a. the suit of diamonds,
b. the suit of spades, and
c. the Ace of spades.

Further, from Figure 3, the probability of a number of mutually exclusive (or disjoint) events can be seen as the sum of the probabilities of the individual events, thus the probability of either a spade or a diamond is one-half, or

Pr(S or D) = Pr(S) + Pr(D)

this is the addition rule for mutually exclusive events, and it follows that Pr(\overline{S or D}) = 1 − Pr(S or D)

= 1 − Pr(S) − Pr(D)

= Pr(\overline{S}) + Pr(\overline{D}) − 1

Thus Pr(\overline{S or D}) does not equal Pr(\overline{S}) + Pr(\overline{D}).

It should be noted that only the probabilities of simple events can be so added: von Mises pointed out that the probability of dying in one's 40th year or getting
married in one's 41st year is not the sum of the probabilities of dying in one's 40th year and getting married in one's 41st year, although the alternatives are mutually exclusive.  

It should be further noted that  

\[ \Pr(\overline{S} \text{ or } \overline{D}) \text{ does not equal } \Pr(\overline{S}) + \Pr(\overline{D}) \]

because the events \( \overline{S} \) and \( \overline{D} \) are not mutually exclusive in that the event \( \overline{S} \) contains hearts and clubs which also are contained in event \( \overline{D} \).

Consider the two events; aces and spades which are not mutually exclusive because the ace of spades belongs to each event.

Figure 4: Venn diagram of the two independent events. Aces with area 1/13 (or 4/52) and spades with area 1/4 (or 13/52). The event Ace of spades has an area 1/52 which is equal to 1/13 x 1/4.

\[ \Pr(A \text{ and } S) = \Pr(A) \times \Pr(S) = \frac{4}{52} \times \frac{13}{52} = \frac{1}{52} \]

This is the multiplication rule for independent events, i.e. the events are probabilistically independent.
Because A and S are not mutually exclusive the probability of one or the other is not the sum of the individual probabilities. In fact

\[ Pr(A \text{ or } S) = Pr(A) + Pr(S) - Pr(A \text{ and } S) \]

this is the addition rule for two events which are not mutually exclusive and is a generalized form for the addition rule for mutually exclusive events. Because the intersection of two mutually exclusive events is a null event space \( Pr(A \text{ and } S) = 0 \), the addition rule for non-mutually exclusive events reduce to that for mutually exclusive events. By using the general addition rule it can be seen that

\[ Pr(A \text{ or } S) = Pr(A) + Pr(S) - Pr(A \text{ and } S) = \frac{4}{52} + \frac{13}{52} - \frac{4}{52} \times \frac{13}{52} = \frac{17}{52} - \frac{1}{52} = \frac{16}{52} \]

which is in accord with the number of cards in a deck that satisfy the description ace or spade: there are thirteen spades (including the ace of spades) and three other aces making sixteen cards in all out of the 52 card deck.\(^{72}\)

It should also be noted that

\[ Pr(A \text{ or } S) = Pr(A) + Pr(S) - Pr(A \text{ and } S) = 1 - (1 - Pr(A)) \times (1 - Pr(S)) \]

after Weiner.\(^{73}\) For two events the utility of this alternative formulation is not apparent but as the number of events increases the former notation becomes quite clumsy, for example, for the three events, A, B and C:
\[
Pr(A \text{ or } B \text{ or } C) = Pr((A \text{ or } B) \text{ or } C) \\
= Pr(A \text{ or } B) + P(C) - Pr(A \text{ or } B) \times Pr(C) \\
= Pr(A) + Pr(B) + P(C) - Pr(A) \times Pr(B) \\
- Pr(A) \times Pr(C) \\
- Pr(B) \times Pr(C) \\
+ Pr(A) \times Pr(B) \times Pr(C) \\
= 1 - (1 - Pr(A)) \times (1 - Pr(B)) \times (1 - Pr(C))
\]

The explanation for the necessity for subtracting the area of the intersection of A and S lies with the fact that the area of the union of A and S is the sum of the areas of A and S less the area of the intersection as shown in figure 5 below.

**Figure 5:** The area of the union of A and S is less than the sum of the areas of A and S by an area equal to the area of the intersection of A and S.

A further note on Venn diagrams is that as the number of events increase it may be easier to represent the events
by rectangles rather than as circles as above. For example figure 4 representing the two events A and S could be redrawn as in figure 6 below.

![Diagram showing Ace of Spades (A ∩ S)](image)

**Figure 6:** The equivalence of Figure 4 (right) redrawn on the left with the events shown as rectangular rather than circles.

In fact, the rectangular format lends itself to a finely detailed event space of each event (52 in total) in the deck of cards as in Figure 7.

![Diagram of Venn diagram of the 52 events, each corresponding to a single card in the event space of a complete deck of cards and each event having area 1/52.](image)

**Figure 7:** Venn diagram of the 52 events, each corresponding to a single card in the event space of a complete deck of cards and each event having area 1/52.
The rules can be generalized for any number of events: $e_1$, $e_2$, $e_3$, ..., $e_n$ thus

$$\Pr(e_1 \text{ or } e_2 \text{ or } ... \text{ or } e_n) = \Pr(e_1) + \Pr(e_2) + ... + \Pr(e_n)$$

for $n$ mutually exclusive events\(^{74}\).

$$\Pr(e_1 \text{ and } e_2 \text{ and } ... \text{ and } e_n) = \Pr(e_1) \times \Pr(e_2) \times ... \times \Pr(e_n)$$

for $n$ independent events\(^{75}\), and

$$\Pr(e_1 \text{ or } e_2 \text{ or } ... \text{ or } e_n) = 1 - (1 - \Pr(e_1))(1 - \Pr(e_2))$$

$$... (1 - \Pr(e_n))$$

for $n$ non-mutually exclusive events\(^{76}\).

That the product rule is only applicable to independent events was exposed in the notorious Collins\(^{77}\) case where the college mathematics instructor considered the following events as independent:

- yellow automobile and interacial couple in car
- black man with beard and man with moustache
- black man with beard and interacial couple
- girl with blond hair and interacial couple
- et cetera. Such properties attributed to the Defendant are clearly not independent of each other\(^{78}\).

Similarly, and more subtly, Dr. Malcolm Simons, admittedly in a lecture restricted to a description of the basics of blood typing, multiplied gene frequencies together to obtain haplotype frequencies\(^{79}\). Yet, as measured by Simons and Tait the haplotype frequencies only approximately equate to the product of the individual gene frequencies, for example\(^{80}\).
frequency of the HLA A1 gene: 0.1653
frequency of the HLA B8 gene: 0.0987
expected frequency of the HLA A1 - B8 haplotype assuming frequencies independent of each other: 0.1653 x 0.0987 = 0.0163
observed frequency of the HLA A1 - B8 haplotype: 0.0698

the two genes are thus strongly linked and exhibit linkage disequilibrium such that the two genes are observed together in greater numbers than the product of their individual observed frequencies would suggest.

A similar but reversed linkage disequilibrium can be seen in the observed frequency of the HLA A2-B8 haplotype (0.001) which is less than the product (0.025) of the observed frequencies of the individual genes A2 (0.2504) and B8 (0.0987). A graphical representation of the lack of independence of these genes might be as depicted in figure 8 below.

Figure 8: Venn diagram showing the lack of independence in the frequency distribution of the HLA A1, A2 and B8 genes.

Were the observed frequencies of these three genes independent an expected graphical representation would be as in figure 9 below.
Similarly Tribe describes the decision of a Swedish court which heard evidence from a parking officer alleging the defendant had permitted his vehicle to remain parked for a period in excess of the permitted time. The Defendant gave evidence that he had driven away and upon returning was able to regain the same parking position he had previously vacated. The court, in assessing the credibility of both witnesses, calculated the probability of the defendant in returning to the position with his two wheels in the same radial position as previously (the officer had testified to the radial position of the defendant's vehicles wheels prior to and after the commission of the alleged offence) as 1/144 (= 1/12 x 1/12). The court declined to convict the defendant on the basis that it was not satisfied to the required degree that the defendant had committed the offence. Besides the issue of the credibility of the parking officer who apparently noted the relative wheel position for two wheels of each vehicle he inspects there are two observations that can be made regarding this case:

Figure 9: Venn diagram showing the expected frequency distribution of the A1, A2 and B8 genes and the A1-B8 and A2-B8 haplotypes if the respective distributions were independent.
(i) the probability value of 1/12 was presumably derived from the hour positions of a clock. The division of the circumference of a circle into twelve equal sectors is not as distinct as the division of the event space of a six-faced die (for example) into six distinct and mutually exclusive events. Further, were it not for the mathematical and chronological accident of arbitrarily dividing the length of the day into 24 equal periods the parking officer may have been restricted to a convention of perhaps only 20 "hours" in the day and consequently a clock face of only ten equal sectors. Presumably a more diligent officer could have noted the wheel positions more accurately: half hour divisions would have permitted the court to infer a probability of 1/24 that the motorist was able to regain his previously vacated parking spot with one wheel in the same radial position as it had been prior to vacating the parking space. Similarly divisions corresponding to one minute would have permitted the court to infer a probability of 1/60.

(ii) the probability value of 1/144 is founded on the premise that the revolution of each of the vehicle's two wheels are independent. Certainly the two wheels do not traverse the same distance. The two wheels are neither independent nor are they fully dependent. The partial dependency does not reduce the probability of their coincidence to 1/12. A stopped clock is
wholly independent of a running clock and yet there will be a coincidence between both clocks once every twelve hours. Yet a clock which loses one minute per day will coincide with a correctly running clock once every 720 days or approximately every two years. A similar fallacy is displayed by Kotz and Stroup in their analysis of the problem of the chances that two clocks will display the same time which is based upon an event space of 720 (= 12 x 60) minutes with any one minute possessing equiprobability with any other minute. It is submitted that the distribution of displayed time of a number of clocks will tend to be weighted towards the correct time.

The arithmetic rules for conditional probability and the graphical representation will be considered in the next section infra.

2.4 Conditional Probability

If you are tired of not being provided by nature, not being physically existing and being miraculous and conventional at the same time, apply for British citizenship. Roughly speaking, there are two possibilities: it will be granted to you, or not.


All probability values are subject to constraints or conditions. Thus, the statement that the probability of drawing a spade from a deck of cards is 1/52 is conditional
on the deck of cards being a standard deck of 52 cards with four suits each of thirteen cards.

Consider the example of Hamburg's wherein a sample of 1,000 persons are classified by sex and product preference as shown by the distribution table below.

<table>
<thead>
<tr>
<th>Male (M)</th>
<th>Prefer Product ABC (B₁)</th>
<th>Female (F)</th>
<th>Prefer Product XYZ (B₂)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>300</td>
<td>500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>400</td>
<td>500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>300</td>
<td>700</td>
<td>1,000</td>
<td></td>
</tr>
</tbody>
</table>

From this distribution the appropriate Venn diagram can be constructed thus:

![Venn Diagram](image)

Figure 10: Venn diagram representing sex and product preference of a sample of 1,000 persons.

From the previous section the probability of preference for product ABC is the ratio of the area of the event representing product preference ABC to the total area. In this instance the probability of preference for product ABC is 0.3.
It may be desirable to determine the probability that an individual prefers one particular product when it is known that the particular individual is male. Thus the probability of a preference for product ABC given that the preference is that of a male is denoted \( \Pr(B_1/M) \). \( M \) and \( B_1 \) are the two events in the sample space and the conditional probability of \( B_1 \) given \( M \) is

\[
\Pr(B_1/M) = \frac{\Pr(B_1 \text{ and } M)}{\Pr(M)} \quad \text{where } \Pr(M) > 0
\]

from the distribution table (or the Venn diagram):

\[
\Pr(B_1 \text{ and } M) = 0.2 \text{ and } \Pr(M) = 0.5
\]

thus \( \Pr(B_1/M) = 0.2/0.5 = 0.4 \).

The imposition of the condition \( M \) restricts the sample space, in this case to a population of 500 males. Thus the Venn diagram could be drawn with a restricted sample space as below (Figure 11):

```
\begin{center}
\begin{tikzpicture}
\draw (-1,-1) rectangle (2,2);
\node at (0,0) {M};
\end{tikzpicture}
\end{center}
```

Figure 11 : Venn diagram representing the product preference of a sample of 500 male persons;

\[
\Pr(B_1) = \Pr(B_1/M) = 0.4
\]

Unless there is a chronological restriction on the order of the events it is possible to determine the conditional probability that an individual is a male given that the individual prefers a particular product. That
is

\[ \Pr(M/B_1) = \frac{\Pr(M \text{ and } B_1)}{\Pr(B_1)} \quad \text{where } \Pr(B_1) > 0 \]

\[ = \frac{0.2}{0.3} = \frac{2}{3} \]

The imposition of the condition \( B_1 \) restricts the sample space (in this case to a total of 300 persons) and the Venn diagram is drawn with the restricted sample space as below (Figure 12):

\[ \Pr(M) = \Pr(M/B_1) = \frac{2}{3}. \]

Thus a conditional probability merely requires a modified Venn diagram so that the sample space under consideration complies with the imposed condition.

As already noted, the lack of a chronological restriction on the order of the events permits an inverse conditional relationship to be inferred

\[ \Pr(B_1/M) = \frac{\Pr(B_1 \text{ and } M)}{\Pr(M)} = \frac{\Pr(B_1 \text{ and } M)}{\Pr(B_1)} \times \frac{\Pr(B_1)}{\Pr(M)} \]

\[ = \Pr(M/B_1) \times \frac{\Pr(B_1)}{\Pr(M)} \]

\[ = \frac{\Pr(M/B_1) \times \Pr(B_1)^2}{\Pr(M)} \]
thus it is possible to assess inverse conditional probabilities from a known conditional probability. This inverse relationship is known as Bayes' Theorem or rule after the Reverend Thomas Bayes (1702-61) who is credited with its formulation. In fact Bayes did not formulate the theorem although it is perhaps fitting that he be so honoured as his work was among the first to consider the inductive probabilities of events. He considered the problem of the "correctness" of attributing the characteristics of a sample to that of the parent population much as the present day opinion pollster or quality control sampler does today. In the context of a lottery Bayes formulated mathematical assessments of the probability that a bystander would be correct if the bystander inferred from noting that of eleven draws in a lottery, ten were blanks and one carried with it a prize and concluded that the ratio of blanks to prizes in the total of the lottery was ten to one.

The work of Bayes was little considered until the early twentieth century when Fisher, Pearson and Neymann "founded" the discipline of mathematical statistics. Whereas Bayes calculated the probability that his sample correctly represented the parent population, the early statisticians approached the same problem from another direction and standardized the acceptable probabilities - they formulated the problem thus:
How many of the parent population must be sampled to allow the sampler to conclude that the sample reflects the characteristics of the parent population with a known probability that the sampler's conclusion is correct.  

Thus, in the context of the electoral pollster Bayes's work provides an assessment of probability that the pollster will be correct if he concludes that the opinions of the polled 2,000 electors reflect the opinions of the parent population. The early statisticians sought to determine the number of electors who should be polled such that the probability of drawing a correct conclusion that those polled reflected the opinion of the parent population would be in excess of (say) 0.975.

Both approaches are inductive in that they seek to generalize from the smaller sample to the larger population from which the sample is drawn. A deductive analysis permits an assessment of the characteristics of the smaller sample to be made from the known characteristics of the parent population.

The responsibility and credit for the formulation of Bayes' Theorem is unknown. The theorem permits a modified assessment of probabilities with the acquisition of further knowledge as described by Weaver:

Suppose you have a closed box containing a large number of black and white balls. You do not know the proportion of black to white but have reason to think that the odds are two to one that there are about equal numbers of black and white balls. You reach into this box, take out a sample of balls, and find that three-fourths of the sample are black. Now before taking this sample you tended strongly to think that the unknown mixture was half white, half black.
After taking the sample you clearly should change your thinking and begin to lean towards the view that black balls outnumber the white in the box.\(^3\)

Consider the example provided by Hodges and Lehmann,

The authorities of a college are considering giving a diagnostic test to the entire student body in order to identify those students who have a certain infectious disease. It is known that the test gives some positive reactions, and the plan calls for subjecting all students with positive reactions to an expensive clinical examination to determine whether they do in fact have the disease. Supposing that previous studies of the test indicate that it gives a positive reaction to about 80% of the persons having the disease, and to about 10% of the persons who do not. Suppose further that general experience with the disease suggests that its incidence among college students is about 1%. From the point of view of a student who takes the diagnostic test and gets a positive reaction, the interesting question is: 'How likely am I to have the disease?' \(^4\)

The relevant probability is provided by Bayes' Theorem:

\[
Pr\left(sick/\text{positive}\right) = \frac{Pr\left(\text{positive/sick}\right) \times Pr\left(sick\right)}{Pr\left(\text{positive/sick}\right) \times Pr\left(sick\right) + Pr\left(\text{positive/healthy}\right) \times Pr\left(\text{healthy}\right)}
\]

\[
= \frac{0.8 \times 0.01}{0.8 \times 0.01 + 0.1 \times 0.99} = 0.075
\]

This conditional probability can be shown on a Venn diagram thus (see Figure 13 below)
Figure 13: Venn diagram representing the student body and the results of the diagnostic test performed on them. Area 1 represents sick students whose condition is not discovered by the test (False negative). The represented area is 0.002. Area 2 represents those sick students disclosed by the test; the area is 0.008. Area 3 represents healthy students wrongly diagnosed (false positive) as sick (Area = 0.099) and Area 4 (= 0.891) represents healthy students correctly diagnosed by the test.

Because only the conditional probability of being sick given the diagnostic test gives a positive result is required the Venn diagram can be redrawn as in figure 14 below.

Figure 14: Venn diagram representing the sample space (those students registering a positive reaction to the test) and the two disjoint events: healthy and sick. The ratio of the area of sick students to the sample space is 0.008 : (0.008 + 0.099) or 0.075.
Alternatively the probability can be expressed as an "odds ratio" of $0.008/0.099 \approx 4$ to $5$. 96

This discussion of Bayes' Theorem is based on two exclusive and disjoint events: sick and healthy students. It can be extended to any number (n) of exclusive and disjoint events:

$$P(e_1/f) = \frac{P(f/e_1) \times P(e_1)}{P(f/e_1) \times P(e_1) + P(f/e_2) \times P(e_2) + \ldots + P(f/e_n) \times P(e_n)}$$

The utility of conditional probability and Bayes' Theorem (inverse probability) will be further discussed in chapter 4.2 "Corroboration" infra in relation to the combined evidentiary value of two or more items of evidence.
Notes to Chapter 2 : Mathematical Theory


2. The Principle of Indifference infra.


7. Ibid.


13. Ibid. 17.


15. i.e., there must be an outcome in that the coin will land with one face up : Halstead L.J., An Introduction to Statistical Methods (1965) 5.


18. Supra n.16.


22. Eggleston (1978) op. cit. 12; Eggleston (1983) op. cit. 11.


24. Ibid.


26. Ayers (1965) op. cit. 47. Accompanying Ayer's discussion is a cartoon showing the traveller strolling across country following none of the three paths.


30. Ibid. at 379-80.

31. The report indicates that such a test was not performed on Miss B, ibid. 379.
32. Mr S, *ibid.* 380.
34. *Supra* n.14.
37. There was evidence in *Stephenson* that Mr S was a part-owner of the Fiat: [1976] VR 376, 379.
38. (1979) 23 ALR 345.
41. *Supra* n.1.
46. Jeffreys *op. cit.* 659.
49. Jeffreys *op. cit.* 659.


55. Supra notes 23, 24.


57. Ibid. 48-9.

58. Ibid. 45.


66. Venn J., Symbolic Logic (1881) ch. 5.


68. Kotz and Stroup op. cit. 14-5; Eggleston (1977) op. cit. 13; Hamburg op. cit. 61; Hodges and Lehmann op. cit. 33; Jeffreys op. cit. 658.


74. Hamburg *op. cit.* 63; Hodges and Lehmann *op. cit.* 35.


76. *Supra n.73.*


82. *Supra n.80.*


88. Hamburg *loc. cit.*; Hodges and Lehmann *loc. cit.*; Kotz and Stroup *loc. cit.*


92. Hodges and Lehmann *op. cit.* 114.


94. Hodges and Lehmann *op. cit.* 114-4, similar examples are provided in Hamburg *op. cit.* 78-80; Kotz and Stroup *op. cit.* 47.

95. Hodges and Lehmann *op. cit.* 114-5.

96. Eggleston (1983) *op. cit.* 204; Finkelstein *op. cit.* 89.

97. Hamburg *op. cit.* 81; Hodges and Lehmann *op. cit.* 116.
3. PROBABILISTIC DECISION MAKING

3.1 Statistical Levels of Confidence

The stark resulting phenomenon here was that somehow or other, despite the fact that over 5% of the slips were yellow, no Negro got onto the panel of 60 jurors from which Avery's jury was selected. The mind of justice, not merely its eyes, would have to be blind to attribute such an occurrence to mere fortuity.

per Frankfurter J., Avery v. Georgia 845 US 559, 564; 97 L.ed. 1244, 1249 (1952)

Very often, when a statistical result is said to be significant, what is meant is that it deviates from chance in the sense that it fails to accord with a priori probabilities.


The normal distribution curve is of immense importance in sampling theory since it may be shown that the mean and standard deviations calculated from random samples tend to be normal if the samples are large\(^1\), even if the population from which the samples are drawn is not itself normally distributed\(^2\). For example, the results of tossing a coin are distributed binomially but for ten or more observations or trials the normal distribution is sufficiently approximate\(^3\).

The area below the normal distribution curve can be calculated and it is a property of the curve that 68% of the total area of the curve is contained within one standard deviation of the mean and 95% of the area is
contained within two standard deviations of the mean. The standard deviation for a number of observations is a measure of the "spread" or dispersion of the observations. Whether the dispersion of the observations (and hence the standard deviation) is large or small, the property of the curve in that the area contained within one standard deviation of the mean always remains the same.

The corollary of the fact that 95% of the area under the normal curve lies within two standard deviations of the mean is that five percent lies outside two standard deviations of the mean. Further, because the curve is a measure of frequency of observations it can serve as a probability measure. Thus, observations that lie outside two standard deviations have a probability of five percent or less. It is possible to postulate that such observations, with probability less than five percent, do not belong to the parent population and are consequently not the results of a random sampling of observations. The statistical test can be formulated thus: The assumption or hypothesis that the observations correspond to a random probability model is rejected whenever the probability of selecting, at random, the observed value is less than a certain critical value (in this case 0.05).

The certain critical value is arbitrary in that it is set by the statistician. 0.05 is a commonly used value as are 0.01 and 0.005. Thus a statistician will specify the critical value or level of significance upon which
his conclusions are drawn. It is possible that an observation is significant at the 0.05 value and insignificant at the 0.01 value. An example cited by Kotz and Stroup is the Baltimore Housing Survey which addressed the effect of public housing on health. The study over three years considered two groups, one living in the slums of Baltimore and the other living in a new public-housing facility. The respective mortality rates were 67% in the slums and 18% in the public-housing facility. Because the observed outcome could be attributed to chance alone with a probability of 0.04 the authors concluded that the survey results were insufficiently significant to conclude that public housing was better than slum dwellings. Had the significance level been set at 0.05 the results would have been significant.

The level of significance can be related to the risk or consequence of error. The mountain climber may expect a higher standard of quality control in the testing of the rope upon which he will entrust his life than will the bank manager who merely wishes to delineate a queue area for his customers.

The origin of the commonly used 5% value is apparently lost in recent antiquity. It may well be because it represents the area outside the two standard deviations or it may be that the 5% governed the choice of two standard deviations from the mean. Dr. Selvin sought
the origin of the 5% level of significance with the following result:

The 5% number that's in the agricultural literature and the education literature and everywhere comes from the very early days of statistics. I was at a seminar a number of years ago where E.S. Pearson, the son of Karl Pearson, and Professor Neyman were at the same seminar. E.S. Pearson was going to talk about frequency distributions. The audience wouldn't let him talk about anything but the history of statistics.

I think Professor Neyman was 80 at the time and E.S. Pearson was 82 or so. They had seen the entire history of statistics between them. E.S. Pearson's father started biometry. Professor Neyman worked with R.A. Fisher. They were all at the London School in the twenties. I was so bold as to say to Mr. Pearson, "Where did this 5% cut-off point come from?" And he said, "Oh, well, Mr. Neyman knows." And Mr. Neyman said, "Oh, I don't know. Mr. Pearson should know." Mr. Pearson said, "I don't know" and said, "Do you think R.A. Fisher made it up?" and Professor Neyman said, "Well, maybe."

Such statistical tests of significance have little or no value unless they are applied to a number of observations. A double-headed penny would not excite the attention of a statistician until the fifth throw of a head (with probability 1/36 or 0.028) suggested to him that such results were not on the basis of chance alone. One turn at the roulette wheel will produce a result with a probability of 1/38 or 0.026 in Las Vegas and 1/37 or 0.027 in Monte Carlo. Such a result, with probability less than 0.05, would not excite the attention of a statistician. Should a statistician wish to test a hypothesis that a coin or roulette wheel was biased he would endeavour to construct a fair test. In the case of the coin he would require 2,500 tosses and in the case
of the Las Vegas roulette wheel, at least 256 trials to permit him to conclude that any estimate of frequency he made as a consequence had a probability of 0.95 of being within two standard deviations of the correct probability/frequency ratio\(^9\). Having determined the sample size of his test he would then conduct it noting the results. In the case of the coin less than 1,201 heads (or tails) or more than 1,299 heads (or tails) would permit him to conclude that the coin is biased because he would be forced to reject the hypothesis that the coin is fair at the 95% significance level. In the case of the roulette wheel, should any number come up less than twice or more than eleven times he may reject the hypothesis that the wheel is unbiased at the 95% significance level\(^10\).

Conversely, the result of a single observation or trial has little or no statistical significance. A coin is not rejected as biased after a single toss merely because the result of the single toss does not exhibit the equiprobability expected between its two faces. Thus, while a frequency statistic cannot serve as a probability estimate unless a sufficiently large number of trials are conducted (The Law of large numbers, \textit{supra}), a decision regarding the classification of a single observation as probable or improbable and hence whether it is statistically significant or not cannot properly be made.
3.2 Sampling with Replacement: A Fallacy

Kissinger's concern about a Russian attack on China was expressed many times. I used to tease him about his use of percentages. He would say there was a 60 percent chance of a Soviet strike on China, for example, and I would say, "Why 60, Henry? Couldn't it be 65 percent or 58 percent?"


An interesting correspondence took place in 1693 between Samuel Pepys, author of the famous Diary, and Isaac Newton, in which Pepys posed a probability problem to the eminent mathematician. The question as originally stated by Pepys was:

A has six dice in a box, with which he is to fling a six
B has in another box 12 dice, with which he is to fling two sixes
C has in another box 18 dice, with which he is to fling three sixes
(Question) – Whether B and C have not as easy a task as A at even luck?

In rather flowery seventeenth century English, Newton replied and said, essentially, "Sam, I do not understand your question."


Consider Cohen's paradox of the gatecrasher. The intractability of the paradox was used by Cohen to justify the abandonment of Negation and Mathematical probability. The failure of mathematicians to resolve the 'lottery paradox' except with *ad hoc*, unprincipled and unconvincing solutions was further justification for Cohen to abandon Mathematical probability. Negation or complementation is the property attached to an arrangement or scheme of disjoint or mutually exclusive outcomes, one of which must occur. Thus with a coin, given that the probability of throwing a head is $P$, the probability of throwing a
tail is 1-P because of the requirement that the probability of throwing either a head or a tail is 1, that is, a certainty.

While Cohen's gatecrasher paradox can be dismissed as "abnormally artificial" it is worthy of analysis if only to demonstrate the ease with which probability theory can delude. The paradox is here set out from Cohen's The Probable and the Provable:

...it is common ground that 499 people paid for admission to a rodeo, and that 1,000 were counted on the seats, of whom A is one. Suppose no tickets were issued and there can be no testimony as to whether A paid for admission or climbed over the fence. So by any plausible criterion of mathematical probability there is a 0.501 probability, on the admitted facts, that he did not pay. The mathematicist theory would apparently imply that in such circumstances the rodeo organizers are entitled to judgement against A for the admission-money, since the balance of probability (and also the difference between prior and posterior probabilities) would lie in their favour. But it seems manifestly unjust that A should lose his case when there is an agreed mathematical probability of as high as 0.499 that he in fact paid for admission.

Indeed, if the organizers were really entitled to judgement against A, they would presumably be equally entitled to judgement against each person in the same situation as A. So they might conceivably be entitled to recover 1,000 admission-moneys, when it was admitted that 499 had actually paid. The absurd injustice of this suffices to show that there is something wrong somewhere. But where?

Cohen is correct in describing as an absurd injustice a situation where 1,000 defendants are successfully sued when 499 had admittedly paid their admission-money. Such a result flies in the face of probability theory which is bound to allocate a probability value of zero to the possibility of there being 502 or more gatecrashers among
a finite population of 1,000 persons made up of 499 paying patrons and 501 gatecrashers. This point was also made by Tribe when discussing the blue bus case: the company owning four-fifths of the blue buses, however careful, would have to pay for five-fifths of all unexplained blue bus accidents\textsuperscript{17}. Perhaps the 502nd defendant in Cohen's paradox can successfully defend the action against him by resort to probability theory that has hitherto been successfully invoked against his predecessors.

A possible solution is for all 1,000 defendants to be joined with the suggested result being that each of the 1,000 defendants being held liable in damages to the extent of 501/1000ths of the entrance money. This is the pragmatic solution which prevents the plaintiff from recovering more than the actual loss suffered\textsuperscript{18} and, it is suggested, would be the solution offered by the statistician. However it is patently absurd that 499 defendants known to have paid for admission should be penalised by paying further in the way of reduced damages. Whether this is preferable to selecting 501 defendants at random and holding them liable for the full admission price is open to question. This course of action contains a latent absurdity in that, given a population of 1,000 persons, there are approximately $10^{300}$ possible permutations in choosing a sub-group of 501 supposed gatecrashers\textsuperscript{19}, only one of which will in fact be the required correct sub-group of 501 gatecrashers. Thus the odds are almost a certainty that at least one and possibly more honest
patrons will be included in such a sub-group chosen at random. For example, there are approximately 125,000 possible permutations in which the sub-group of 501 is made up of all the paying patrons (499) and two gatecrashers. The odds of choosing such a sub-group are then about 125,000 times greater than those of choosing the correct sub-group of gatecrashers. Such a course of action will penalise the correct number of defendants but will almost certainly impose an injustice on some of them. Is this any more tolerable than holding an incorrect number of defendants liable for reduced damages?

Of course, the above discussion is irrelevant because the paradox as analyzed by Cohen contains a fallacy. The plaintiffs, in choosing their defendants at random are bound to choose without replacement. Consider a deck of cards. What is the probability of drawing at random a specified card? It is 1/52. What is the probability of drawing, at random and on the second drawing, the same card? If the drawn card (from the first drawing) is replaced in the deck prior to the second drawing it is again 1/52. If however, that card is not replaced the probability of drawing the specified card on the second drawing is dependent on the "identity" of the card first drawn. If the card first drawn was actually the specified card, then the probability of again drawing the specified card on the second drawing is zero, it being impossible to draw from a pack of cards a card that is no longer in the pack. If the card first drawn was not the specified
card, the probability of drawing it on the second draw is not 1/52 but 1/51 because the drawing is from a deck which now only contains 51 cards.

Consider the gatecrasher problem afresh. The plaintiffs initiate suit against A and given the premise that the civil standard of proof borne by the plaintiffs is discharged by showing that the probability of A being a gatecrasher is 0.501, it can be concluded that the plaintiffs will succeed against A. Flushed with success, the plaintiffs initiate suit against B. What is the probability that B is a gatecrasher? Of the total population of 1,000 persons there are 499 honest paying patrons

500 dishonest gatecrashers yet to suffer justice

and one gatecrasher (A) who has suffered justice. The probability that B belongs to the sub-group of gatecrashers who have suffered justice is zero because that sub-group contains only one member (A). Thus the probability of B being a gatecrasher is 500/(499 + 500) or 500/999. Presumably B will lose the case because the balance of probabilities favours the plaintiff's case. Consider now the third defendant C. The probability that C is a gatecrasher is 499/998 or even odds. The plaintiffs will fail to prove their case and will not succeed against C. There is of course the possibility that A, being an honest paying patron has unjustly lost his case. However he is bound by the decision of the court: it has been found as legal fact that A is a gatecrasher.
The analysis offered above would appear to be no better than that provided by Cohen: his conclusion allowed for 499 to unjustly lose their cases while the above analysis allows 499 dishonest gatecrashers to escape justice. It would seem that the rodeo promoters can only recover two entrance-moneys and no more. It is now possible to echo Cohen and ask: the absurd injustice of this suffices to show that there is something wrong somewhere. But where?

It may be that our system of law and its civil standard of proof is responsible for the absurdity although it is not suggested that these be changed. Can it be the manner in which probability theory has been applied? Again, it is suggested that this is not the case. It is submitted that the fault lies in expecting more from probability theory than it is capable of providing.

Consider again the defendants A and B whose misfortune arises from the purely fortuitous fact that they were prosecuted first. Williams in his discussion of this paradox wrote:

Evidently, statistics cannot make good a deficiency of evidence involving a particular defendant. The true reason why the proof fails in the gatecrasher case and the Blue bus case is that it does not sufficiently mark out the defendants from the others. After introducing his own hypothetical case illustrating this point with two equally likely defendants in a criminal action, only one of whom is guilty, Williams continues:

...they would both be acquitted, not both convicted. Our sense of justice requires evidence to be given singling out the defendant from other possible culprits.
This requirement that evidence should focus on the defendant must be taken to be a rule of law relating to proof, distinct from the general rule governing quantum of proof.21

Sir Richard Eggleston, in his discussion of the paradox and criticism of Williams's solution, questioned the existence of the rule of law relating to proof as suggested by Williams, there being no authority for the existence of such a rule.22 Since the discussion and the publication of the second edition of Sir Richard's text the Supreme Court of South Australia has considered the issue in a civil suit with all three members of the bench deciding in accordance with the analysis given by Williams and the Chief Justice expressly adopting Williams's discussion.23

Further, in Williams's hypothetical criminal case each equally likely accused can point to the existence of the other as being sufficient to raise a reasonable doubt and thus forestall the prosecution proving their case to the standard required to justify conviction. Consider the use of probability theory and statistics made by the nuclear physicist. The physicist is able to confidently inform us that in a certain period of time, defined as the half-life of an element, fifty per cent of a radio-active element will suffer spontaneous decay. What the physicist cannot do is tell us which atoms will decay. The physicist is not particularly concerned with this question and in his field probability theory is sufficient for his purposes. The physicist is not concerned with individual identity; the jurist should be.
The lottery paradox is a paradox in the truest sense
and does not justify the abandonment of mathematical
probability theory. Given a lottery with a large number
of tickets \( n \), each ticket holder has only a probability
of \( 1/n \) of winning the prize where \( 1/n \) is a small probability.
The paradox is that notwithstanding the small probability
that each of the ticket holders has, one of the ticket
holders does in fact win the lottery. The arithmetic
rule of addition for the probability of one or another
is (from Section 2.3 supra):
\[
Pr(A \text{ or } B) = Pr(A) + Pr(B)
\]
where the events \( A \) and \( B \) are disjoint. Similarly
\[
Pr(1 \text{ or } 2 \text{ or } 3 \text{ or } 4 \text{ or...or } n) = Pr(1) + Pr(2) + Pr(3) + Pr(4) + ... + Pr(n)
\]
and where \( Pr(1) = Pr(2) = Pr(3) \text{ etc. } = 1/n \)
\[
Pr(1 \text{ or } 2 \text{ or } 3... \text{ or } n) = \frac{1}{n} + \frac{1}{n} + \frac{1}{n} + ... + \frac{1}{n}
= n \times \left( \frac{1}{n} \right)
= 1 \text{ (a certainty)}.
\]
It is submitted that this mathematicist solution is
neither ad hoc nor unprincipled and is thus not correspondingly
unconvincing. 

The real reason why probability theory cannot resolve
the problem of the gatecrasher (which is no longer the
paradox described by Cohen) is that the problem is not
amenable to probability theory. That the problem is not
so amenable is not sufficient cause to resort to Cohen's
'Inductive probability' which does not advance the solution
of the problem any further. Indeed Cohen's Inductive
probability merely redefined the probability in favour of the losing litigant: Cohen's interpretation of the mathematics of the gatecrasher problem is that a defendant can lose his case even when the probability that he is an honest paying patron is as high as 0.499. By abandoning negation the defendant still loses his case but he is not permitted to complain of the probability value of 0.499 supporting his honesty. The probability value of the losing party's case is redefined as zero in Cohen's Inductive Probability theory\(^2\).

If a man is hired as a gardener, can his employer complain of his inability to repair the television set? Can the employer complain about the state of the garden after he has locked up the gardening implements in the tool-shed and disallowed the gardener's use of the tools? Reminiscent of the court's reference to mathematics as a sorcerer in the Collins case\(^2\) is the following from a textbook on the use of statistics in the social sciences:

> At first sight, models look like just tools, just techniques, just hired servants to do the house-cleaning jobs. However, before long, the relation develops into something more than a relation between maid-servant and master: the master marries the girl\(^2\).

It is not sufficient to construct a hypothetical fact situation that is likely to bring about an injustice: all that is required is a restriction limiting the available information to that which tends to support an incorrect interpretation of an event. There is little to be gained by postulating an innocent victim of a
miscarriage of justice and then bewailing the inadequacies of a system of justice. While there may be merit in Cohen's Baconian inductive probability thesis it is not demonstrated by his discussion of the lottery paradox or his own gatecrasher paradox.

3.3 Conditional Probability

But as regards the man whose veracity is one-half, we are (as Mr. Monro has very ingeniously suggested) only too well acquainted with such witnesses, though under a somewhat different name; for this is really nothing else than the case of a person confidently answering a question about a subject matter of which he knows nothing, and can therefore only give a mere guess.

John Venn, The Logic of Chance, (2nd. ed. 1876) 442.

Expert opinion, which is only an ordinary guess in evening clothes...


As previously discussed, all probability statements are subject to conditions expressed or implied and such conditions impose a restriction upon the sample space under consideration. An extension of conditional probability where the condition is expressed is Bayes's Theorem wherein assessments of probability values may be modified with further available information. Weaver has cautioned that the theorem is of limited usefulness because it is not capable of creating a judgement out of nothing: whereas the theorem furnishes a basis for modifying a prior opinion, it is powerless to originate an opinion because
[It can only take a previous judgement (an *a priori* opinion, as the professionals say) and tell you how you are justified in modifying that opinion on the basis of the new evidence. In very many practical circumstances it is difficult or impossible to start out, at the very beginning, with any reasonable or defensible *a priori* opinion. And in that case, Bayes's theorem cannot build a new judgement, because it has nothing on which to build.]

Weaver concludes that the problem inherent in the use of the theorem has led some extremists to throw the theorem entirely away whereas a more reasonable procedure is to use it with caution, understanding and restraint.

Hamburg suggests that the *a priori* probabilities are founded on subjective judgements, intuitions and present quantitative knowledge.

In fact, Bayes's Theorem can be utilised in "two-stage" tests where a broad (and perhaps inexpensive) survey or sample yields an initial assessment of probability value to be followed up by a more detailed survey.

It is an artifice to take two individual items of knowledge or evidence and to construct a two-stage test or assessment of probability. Eggleston concedes that his examples are no better than "guesses" and it would thus seem that an assessment of probability value is founded upon one guess modified by a further guess. Whereas it is arguable whether simple probability theory can be resorted to in applications to non-simple problems such as those human affairs that come before the courts, it is submitted that a compound of simple probability
models is of less utility and perhaps even dangerously misleading. Eggleston suggests that Bayes's Theorem is instructive even when not used quantifiably in that it permits a jury (or other fact finding body) to appreciate how one item of evidence can reinforce and strengthen the evidentiary value of another such item. Conversely, Eggleston suggests that an appreciation of Bayes's Theorem will result in the combined value of the two evidentiary items not being accorded undue weight. It is submitted that mathematical probability theory is hardly required to accomplish the first and that the second may not be realised if the jury is overborne by the grandeur of the science of the mathematics.

Kahneman and Tversky have conducted experiments using as their subjects persons without prior training in Probability theory. They have concluded that such subjects are not capable of assessing the correct weighting to be given to known background information because the subject's subjective assessment of the information is not in accord with Probability theory. Kahneman and Tversky and others have coined the term 'subjective probability' to denote intuitive lay assessments of probability as opposed to those founded on the mathematical theory. It is important to distinguish between the assessment of the probability of a future event occurring or the probability of the unknown outcome of a past event and the known outcome of a past event. The latter is hard fact while the former is conjecture based on admittedly
sound mathematical principles. It is one thing to note that in June, 1950 an unidentified man walked up to a dice table at Las Vegas and made an amazing twenty-eight consecutive passes with the dice and another to compute the odds against such an occurrence to be so large that there is a near certainty that such an event cannot occur; the fact remains that such an event did occur. The mathematical computation is no more than an assessment of the probability of the occurrence of this event.

An example of conditional probability, the source of which is unfortunately unknown but its construction is suggestive of Kahneman and Tversky as its authors, was introduced at a seminar conducted by Mr. Cohen at the Monash University Law School on the fourth of September, 1980. The example can be formulated in several ways:

I  **The Doctor's Dilemma**

There are two fatal diseases, for example Blue measles and Green measles, which both manifest themselves in identical symptoms and for which effective cures are known. The treatment for both diseases differs and the result of treating one as for the other is fatal. Thus, a patient exhibiting the symptoms will, if untreated, die. If treated for the particular strain from which he is suffering, the patient will affect a full recovery. If, however, the patient is suffering from one strain and is wrongly treated for the other, the result is fatal. The patient's
doctor has two further pieces of information available to him. They are

(a) there exists a test which can successfully distinguish between the two strains four out of five times or 80% of the time. On those occasions when the test is unsuccessful it diagnoses the strain incorrectly as that which the patient is not actually suffering from. Thus, if we consider only those patients suffering from the blue strain, we would expect 80% of those patients would be correctly diagnosed as suffering from the blue strain while 20% would be incorrectly diagnosed as suffering from the green strain; and

(b) the known prevalence of the disease is in the proportion of blue to green as is 85:15, i.e., given a number of patients displaying the symptoms we would expect 85% will be suffering from the blue strain and 15% suffering from the green.

What is the doctor to do? If he treats his patients according to the test results the doctor's patients will suffer a 20% failure rate or, more optimistically, the doctor will enjoy an 80% success rate. If however, the doctor ignores the test results and treats all his patients for the blue strain he will achieve the higher success rate of 85%. This follows from basic mathematical probability founded on the two statistics
of the 85 to 15 prevalence ratio and the 80% successful diagnosis rate of the clinical test. Yet, a lay patient given these facts would opt, it has been suggested, for the treatment indicated by the test. It is the lay patient's assessment of the facts and the conclusion drawn by him that is termed 'subjective probability'.

II The Taxi Accident

A town has a taxi fleet which is made up of 85% blue taxies owned the Blue Taxi Co. and the remaining 15% are green and owned by the Green Taxi Co. There is civil litigation arising out of an accident involving a taxi and the only witness to the accident is known to be 80% reliable. By this it is meant that in four instances out of five requiring colour identification the witness will correctly identify the colour of the taxi (as blue or green as the case may be) and in the remaining instance will incorrectly identify the colour (as green or blue respectively). In the case under consideration the witness has testified that the taxi involved in the accident was green. Unlike the 'Doctor's Dilemma' where the problem was postulated to be subject to an 85 to 15 prevalence ratio, the 'Taxi accident' depends on an assumption that because there is an 85 to 15 prevalence ratio in the taxi population, it follows that there will be an 85 to 15 prevalent ratio in the number of accidents suffered by the two taxi companies.
Given this assumption it can be shown that the probability of the taxi involved being blue is greater than that of it being green notwithstanding the witness's testimony that the taxi was green. The probability values are:

- that the taxi is blue and correctly identified by the witness: 0.68
- that the taxi is green and correctly identified by the witness: 0.12
- that the taxi is blue and is incorrectly identified by the witness as green: 0.17
- that the taxi is green and is incorrectly identified by the witness as blue: 0.03

These probability values are shown in a Venn diagram (figure 15):

![Venn diagram](image)

Figure 15: Venn diagram showing the sample space of the town's taxi fleet and the 80% reliable testimony of the witness.

Given that the witness has testified that the taxi involved was green the probabilities are:

- 0.586 that the taxi was in fact blue, and
- 0.414 that the taxi was in fact green.
and these probability values are shown on the Venn diagram (figure 16).

Figure 16: Venn diagram showing the sample space subject to the condition that the witness has identified the taxi involved as green.

Thus the mathematics of the problem suggest there is a greater likelihood, in the instant case, that the witness is committing one of his known and expected one-in-five erroneous identifications. What is the decider of fact to do? It has been suggested that lay subjects, asked to consider themselves as jury members in this hypothetical civil suit, tend to cast their lot with the witness and find it was a green taxi that was involved in the accident. Again, this disregard of relatively simple mathematical probability theory is termed 'subjective probability' and is used as a basis for the conclusion that such persons are not to be entrusted with the duty of serving on a jury. A variant of this problem has since been postulated by Cohen as a modification of his 'gatecrasher's paradox': where 85% of those attending a rodeo fail
to pay the entrance charge and one of those attending is chosen at random and sued by the promoter to recover the charge, what is the finder of fact to do when an 80% reliable witness identifies the defendant as a patron who did pay at the gate.

III Bookbag and Poker Chips

The two formulations already given are no more than a colourful dressing up of a traditional illustration for conditional probability. Consider five urns (or bookbags), four of which contain 85 blue poker chips (or marbles) and 15 green chips while the fifth urn contains 85 green chips and 15 blue chips. Choosing an urn at random and from that urn, again at random, choosing a chip the probability of ultimately drawing a blue chip is higher than that of drawing a green chip: 0.71 as opposed to 0.29 (refer to figure 15 supra). The problem can now be reversed. Given that an urn has been chosen at random and from that urn a chip is drawn at random and found to be green; what is the probability that the green chip so drawn came from one of the four urns containing only 15 green chips as opposed to the probability that it was drawn from the fifth urn which contained 85 green chips. Using probability theory it can be shown that there is a higher probability that the drawn green chip came from the fifth urn (with probability 0.586) and not from any of the first four urns (0.414, refer to figure 16 supra).
The three formulations offered are identical and illustrate a phenomenon akin to that described by Tribe as the "dwarfing of the soft variable". Venn uses the expression "a contest of opposite improbabilities". The 85 to 15 ratio of blue to green being sufficient to overwhelm the 80 to 20 ratio of correct to incorrect. In the examples given the prevalence of the blue strain of the disease is equivalent to the known number of blue taxis in the town (or the known number of non-paying spectators at the second of Cohen's rodeos) and both are equivalent to the number of blue marbles in the first four urns in the last example. The clinical test of 80% reliability available to the doctor is consistent with the reliability of the witness in the taxi accident case (or the witness at Cohen's second rodeo) and also the fact that 80% of the urns, i.e. four out of the five, truly reflect the population ratio of marbles. The fifth urn (20% of the five) incorrectly represents the population ratio of marbles and is equivalent to the 20% unreliability of the clinical test and the witnesses. Thus choosing an urn at random and from the urn choosing a marble and finding the marble so chosen to be green is the equivalent of the witness testifying that the taxi involved in the accident was green (or the defendant-spectator at the rodeo is identified by the witness as a paying patron) if it can be assumed that taxi accidents are the result of a random process and that the reliability of the witness is also random. The conclusion that the green marble so chosen is more likely to have been drawn from the fifth urn is the
equivalent of concluding there is a higher likelihood that the witness has mistakenly identified a blue taxi as green.

Is it fair to pose this type of problem to a jury or a group of lay subjects being tested for psychological research? Is it fair to impose such a lay jury on litigants when it has been suggested that such a jury will arrive at an incorrect conclusion from the known facts? The answers to these questions require more than a bold assertion that, following experiments, subjective conditional probability assessments are fallacious. The questions require investigation before any tentative answers can be offered if indeed such answers do exist.

To begin, the three formulations of the problem are not identical. In the context of mathematical probability they are but it is intended to proceed beyond mathematical probability. Consider the Doctor's Dilemma. By reconducting the clinical test of 80% reliability a second time the doctor can achieve a 91% reliable diagnosis. Conducting a second test is not feasible in the taxi accident case because even if the logistics of re-creating the random accident can be overcome, the dishonest taxi company need only ensure that none of their taxis are in the vicinity of the accident scene. Furthermore, to re-create the accident requires the abandonment of randomness while the retention of randomness can either result in the non-occurrence of any accident or the occurrence of an accident that can have no bearing on the accident being considered by the
court. Similarly, the conduct of a second trial using
the urns and marbles, although feasible and easily done,
is of little value. Assuming that on the second trial
a green marble is again drawn. No conclusion relevant
to the first trial can be formed and a similar non-conclusion
would be the result of drawing a blue marble on the second
trial.

Continuing with the third formulation, that of the
urns and marbles, it is possible to make use of hindsight.
That is, after drawing a green marble the remaining contents
of the particular urn can be tipped out and counted to
determine beyond doubt whether the urn was the fifth urn
or one of the first four urns. Valuable as it is, hindsight
is not available to the jury in the taxi accident case,
it not being possible to tip out the contents of the witness
and, by counting his marbles, determine the reliability
of his testimony. Similarly, the doctor can avail himself
of hindsight although such hindsight is of little value
to his patient who has already been irrevocably treated
by the time such hindsight becomes available to the doctor.
The information to be derived from hindsight is not available
to the doctor so as to permit the correct diagnosis prior
to treating his patient.

Thus the doctor, relying on a clinical test that is
80% reliable, can only achieve a lesser recovery rate than
a colleague who treats all his patients for the blue strain
regardless. How can this be so? A partial explanation
is that neither has utilized all the available information. The doctor who treats all his patients for the blue strain has not made use of any information that the clinical test may provide; this deliberate ignoring of the test results is, by definition, incorporated within the problem as it is posed. This may be demonstrated by changing the reliability of the test. Assume that the test is only 50% reliable - the doctor's success rate for treatment is still 85%. Increase the reliability of the test to 99% - the success rate remains at 85%. This is because the doctor's "diagnosis" is independent of the test. Consider now the doctor who prescribes treatment as indicated by the test and suffers only an 80% success rate. This doctor is ignoring a piece of available information: namely the fact that the blue strain is more prevalent than the green in the ratio of 85:15. Should it be doubted that the second doctor has not made use of the known prevalence ratio, those doubts can be dispelled by altering the ratio, for example reversing it to 15:85 - the success rate, based on the 80% reliable test, remains at 80%. Similarly for a prevalence ratio of 50:50 or even 99:1. The doctor who bases his diagnosis on the test cannot rise above the 80% success rate because his diagnoses are founded on only one of two available pieces of information. The pieces of information are used independently by both doctors without regard to the other piece of information. The problem's formulation is one of independent conditional probability. It is possible to illustrate the difference between such a problem and another involving dependent conditional problem by an example.
The facts are the same as for the 'Doctor's dilemma' except that, while the reliability of the test when conducted on blue strain sufferers remains at 80%, the reliability of the test is 90% when administered to those patients suffering from the green strain. That is, testing blue strain sufferers will result in a correct diagnosis 80% of the time and an incorrect diagnosis of the patient as a green strain sufferer 20% of the time. However, testing green strain sufferers would result in a correct diagnosis 90% of the time and an incorrect diagnosis of the patient as a blue strain sufferer 10% of the time. The reliability of the test is dependent upon that which the doctor is seeking to determine: the particular strain suffered. This is an example of dependentconditional probability.

Reconsider the 'Doctor's dilemma' which was formulated as a problem incorporating independent conditional probability. It is possible to show that if the doctor who relies on the clinical test and can so far only achieve an 80% success rate then takes into account the known prevalence of the two strains, the success rate can be increased to equal that of his colleague who treats all his patients alike for the blue strain and has an 85% success rate. This is done by treating all the patients diagnosed as blue strain sufferers accordingly and reclassifying 0.586 of those diagnosed as green strain sufferers as blue strain sufferers. The process is repeated until the doctor has 848 live blue strain sufferers and 2 live green strain suffers who have been successfully treated at the expense
of 148 deceased green strain sufferers and 2 deceased blue strain sufferers who have been unsuccessfully treated.

Whether this end result is preferable to 850 blue patients effecting recovery at the expense of all 150 of the green patients is questionable. The procedure has been outlined only to demonstrate that the doctor who initially uses the results of the test available to him need not suffer a lesser success rate in treating his patients.

Thus far only the mathematics of the problem have been considered. Consider now the subjective probability assessment made by the patient when he elects to undergo treatment in accordance with the test result or the jury members who choose to believe the testimony of the 80% reliable witness. Why should these lay assessments differing from the mathematical assessment be made and are they less valuable than the mathematical assessment? It is submitted that they are not. The patient does not have the benefit of relying on the law of large numbers as does the doctor. For the patient, the decision to abide by the medical test result is a 'once-off' decision. The doctor can rely on Chalmer's fairground operator: What's lost upon the roundabouts/we pulls up on the swings49, while the patient may feel that he has to accept only one of a roundabout or a swing. Similarly the jury members may recognize that as far as they are concerned the taxi accident is a 'one-off' incident. To use the urn and marble illustration,
the patient or the jury member may suspect that for them, each of the five urns contains only one marble and not the hundred that the doctor has in each urn. In cases where large numbers are not involved such as the jury hearing or the personalized trauma of the patient, probability is of little or no value. Take for example the 'once-off' toss of a coin. Suppose it comes down heads. Is the coin to be condemned as biased because it has not displayed the expected equality of likelihood between heads and tails of an unbiased coin?

For another example consider a lottery recently conducted in Victoria designed to provide a total of $1.5m in prize money including a $1m first prize. Before the lottery can be drawn 100,000 tickets at $25 each must be sold. Thus a total of $2.5m. was paid in by purchasers in return for a total payout of $1.5m. The law of large numbers suggests it is more expedient, with less administrative overheads, to simply pay out a prize of $15 to each purchaser of a $25 ticket. As far as the organizers of the lottery are concerned the result would be no different from the lottery as it is presently conducted. The organizers can rely on the law of large numbers. The ticket purchasers, however, are hoping to defy the law and, surprisingly enough, their hopes are not misplaced. The first prize winner of $1m has certainly successfully defied the law of large numbers as has the luckless participant whose ticket fails to secure a prize of any size. The doctor is in the same position as the organizers of the lottery: the identity of the winning ticket-holder is irrelevant to
the lottery organizers while the purchaser is most concerned with the identity of the winning ticket-holder. Similarly, the identity of those who make up the 85% recovery group is of no concern to the disinterested doctor while the individual patient understandably desires to include himself within those patients who effect a recovery.

Consider the 'Doctor's dilemma' from a different viewpoint. Among 1501 of the doctor's patients is a valuable member of society: a concert pianist of world renown or perhaps another Einstein. Assume that this exceptional patient has a value equivalent to 500 'ordinary' patients. What is the doctor to do? The prevalence ratio of 85:15 is now either 1775:225 or 1275:725 depending on which strain of the disease is suffered by the exceptional patient. Is it too far fetched to suggest that from the ordinary patient's narrow and understandably selfish viewpoint he considers himself of greater value than his fellow patients?51

There are four further points to be made before leaving the question of green against blue. The first suggests a contradiction between the problem's formulation and the conclusions that follow from the application of probability theory. The witness is, according to the problem, 80% reliable. Thus the probability of correct testimony is 80% reliable. Thus the probability of correct testimony is 80% irrespective of taxi colour. Yet the conditional probabilities are, given that the witness testifies that the taxi was green, 0.414 that it was in fact green and
0.586 that it was in fact blue. Thus, if the witness testifies that it was a green taxi, the witness's reliability is only 41.4\% and if he testifies that the taxi was blue, his reliability is 95.8\% by a similar calculation. There would appear to be a contradiction between the requirement that the witness's objective reliability is 80\% and the conclusion that the witness's reliability is dependent on the colour that the witness believes was the colour of the taxi in the accident\textsuperscript{52}.

Secondly, the problem's formulation in distributing the witness's reliability over the sample space of all the taxis is an application of the Principle of Indifference - and yet the problem's formulation violates the essential requirement without which that Principle cannot be appealed to: in the absence of any reasons to the contrary, the probability of any one of \( n \) mutually exclusive events is \( 1/n \). That a witness (albeit of only 80\% reliability) testifies that the taxi involved was a green taxi is sufficient cause to require the abandonment of the probability assessment (founded on the Principle of Indifference) that there is a 0.85 probability that a blue taxi was involved:

That in one throw of dice there is a quantitative probability, or greater chance, that a less number of spots than sixes will fall uppermost is no evidence whatever that in a given throw such was the actual result. Without something more, the actual result of the throw would still be utterly unknown. The slightest real evidence that sixes did in fact fall uppermost would outweigh all the probability otherwise\textsuperscript{53}. 
Thirdly, even accepting a quantitative measure of a particular witness's reliability for veracity - assuming that such witnesses go about the world bearing a stamp on their foreheads attesting to their reliability - the witness box is not the normal habitat of such persons. To accept such a quantitative measure of general reliability and apply it to the courtroom testimony of such a witness is no better than relying on actuarial tables of mortality to determine the probable length of life of a soldier who is already in the midst of battle.

The last point is best demonstrated by increasing the prevalence ratio and the reliability of the witness. In the case where the 99% reliable witness testifies that the taxi was green when green taxis make up only 1% of the taxi population, the conditional probabilities are equivocal in that they allocate a 0.5 probability to each colour. Ayer considered the case of Petersen, a Swede who made a pilgrimage to Lourdes last year. 95% of Swedes are Protestants. But 95% of those who make pilgrimages to Lourdes are Roman Catholics. The two statements are flat contradictions if it is desired to assess the odds that Petersen is a Protestant. It is not a contradiction if the problem is interpreted by identifying probability with relative frequency. The answer then becomes a restatement of the facts that Petersen is a Swede who has made a pilgrimage to Lourdes and that 95% of Swedes are Protestants and 95% of such pilgrims are Roman Catholics - there is no contradiction here; but equally the question to be determined about Petersen has disappeared.
The prevalence ratio and the reliability can be increased to 100%. What is to be concluded when the 100% reliable witness testifies to a green taxi when all the taxis in town are blue? Although the suggestion that an out-of-town green taxi be sought may seem trivial, it does highlight an inherent difficulty in the problem - to what finite 'population' do we restrict ourselves when considering the prevalence ratio?

A final example of subjective probability assessment, this time drawn from reality, was provided by Tversky in the discussion following the reading of a paper of his before the Royal Statistical Society:\textsuperscript{57}

The following observation, reported by J.L. Bower of the Harvard Business School, shows that this is not the case [i.e., the observed non-optimality is due to the artificial nature of the situation and that it will surely disappear in a realistic context where the payoffs are substantial].

Fighter pilots in the Pacific during World War Two encountered situations requiring incendiary shells about 1/3 of the time and armour-piercing shells about 2/3 of the time. Since there was no general procedure for predicting on every mission which type of shell would be required, the optimal policy was clearly to use armour-piercing shells on every mission. It was observed, however, that when left to their own devices, pilots armed themselves with incendiary and armour-piercing shells in the proportion of 1 to 2. Thus, the experienced fighter pilots acted much like the naive subjects in the psychological laboratory, even though their own lives were at stake.

Unfortunately further information is missing but on a purely probabilistic analysis Tversky would appear to be correct. A pilot armed in accordance with Tversky's conclusion would be well prepared for 2/3 or 6/9ths of the missions he flew while the pilot who randomly chose
his ammunition subject only to the constraint that the arm himself with armour-piercing shells twice as often as incendiary shells would be well prepared for only 5/9ths of his missions. Another way of evaluating these figures would be to suggest that, given it was fatal for a pilot to find himself in a situation requiring ammunition different from that which he was armed with, Tversky's pilots would average a 'life expectancy' of three missions (failing to return from their third mission) while those pilots who randomly selected their ammunition subject to the one to two proportion constraint would average a 'life expectancy' of only two and one-quarter missions. Thus the criticism offered by Cohen is unfounded, although this is not to concede the validity of Tversky's analysis.

It is not clear from the limited available information whether the experienced fighter pilots armed themselves with only one type of ammunition for each mission and selecting the type on the basis of the one to two proportion constraint or that they armed themselves with a mix of ammunition for each mission; the mix being in the proportion of two armour-piercing shells to one incendiary. Without further information this example, notwithstanding its realistic origin, is of less value than Tversky suggests.

Tversky's fighter pilots is an example of optimum decision making for maximizing the certainty of success at the expense of a known failure rate. Our legal system
does not work that way. The legal system attempts, however imperfectly, to maximize success without the expense of a known failure rate. The fighter pilots, figuratively, were flying in the dark. The court-room analogue of the pilots irrevocably choosing their ammunition before they left on a mission is to instruct a jury to irrevocably decide a case before it came to trial or before any evidence was lead. In this context the company that owned four-fifths of the blue buses, however careful, would have to pay for five-fifths of all unexplained blue bus accidents. If there is a 90% conviction rate in the Victorian courts of summary jurisdiction as suggested by the Chief Commissioner of Police when advocating the reform or abolition of the jury system, then perhaps there is scope for recording a conviction against every defendant without the necessity for an expensive and inconvenient trial. The administrative convenience of abolishing trials is bought with the known incorrect conviction of 10% of the defendants.

3.4 Coincidence

The improbable - by definition being not impossible - sometimes does occur.


A 'coincidence' is 'a notable concurrence of events or circumstances without apparent causal connection'. Only those coincidences that are remarkable are likely to be remarked upon. When a bridge player is dealt the
A, K, 9, 5 and 3 of spades, the Q, J and 4 of hearts, the K and 2 of diamonds and the A, Q and J of clubs the hand dealt is not at all remarkable notwithstanding that the odds against being dealt such a hand by chance (or any other specified hand of thirteen cards) are in excess of 63 to 1. If the same player were dealt all thirteen spades his fellow players may harbour doubts regarding the dealer's honesty. Yet the odds for both hands are the same. Further, an English mathematician suggests that in 1953 approximately two million people in England each play an average of thirty bridge hands a week and concludes that it is not at all surprising that the newspaper reports of a hand of thirteen spades is something of an annual event.

Examples of interesting coincidences abound. A regular correspondent for the Australian Law Journal reported meeting Sir Richard Eggleston in London and on the very next day found himself sitting on the very next seat to Sir Richard on a bus. Of interest in calculating the number of different combinations that can be drawn from a finite population are the Bell numbers, so named after the mathematician who first analyzed them in depth. Gardner notes that in the 1920s and 30s Bell was a prolific writer of science fiction under the pseudonym Taine and in 1951 Taine grasped the opportunity of writing a glowing review of the mathematician Bell's text *Mathematics, Queen and Servant of Science*. In further researching probability theory, this writer stumbled upon the review
by the American attorney Train in the Yale Law Journal of *Yankee Lawyer: the Autobiography of Ephraim Tutt*. Train's review of the book was also laudatory and closed with a paraphrase of Voltaire's aphorism: "If Mr Tutt did not exist; it would be necessary to invent him" which can be compared with Taine's "the last flap of the jacket says Bell 'is perhaps mathematics' greatest interpreter'. Knowing the author well, the reviewer agrees.

Consider Gardner's description of the advertisements which appeared in the *New Yorker* of November 22, 1941. The advertisements, for a dice game, appeared to give notice to Japanese undercover agents in the United States of the impending attack on Pearl Harbour including the date, time and location. The advertisers were investigated by the FBI after Pearl Harbour and their conclusion - only a coincidence. Now consider the *Daily Telegraph* crosswords of May 3, 24 and 31 and June 2, 1944 which puzzles provided clues for the words "Utah", "Omaha", "Mulberry", "Neptune" and "Overlord" - all code words associated with the allied landings on mainland Europe on D-Day. In this case the British MI5 investigated and concluded that the puzzles were innocent coincidences.

Although such coincidences are not causally connected they can be of such a striking nature that it is possible for an undetermined connection to be drawn between such events. When you put a kettle on the fire, there is within the probability laws of thermodynamics, an infinitesimally
small theoretical possibility that the water will freeze rather than boil. But as Eddington remarked, we should never believe that such a thing happened by chance, and would be justified in looking for an explanation beyond the known principles of science\textsuperscript{71}.

Thus, persons unaquainted with the birthday problem\textsuperscript{72} will underestimate the probability that at least two of thirty persons chosen at random will share a common birthday. Such an underestimation reflects a coincidence of striking impact and real insignificance according to Littlewood\textsuperscript{73} or to put it in legal terms - the prejudicial effect of such evidence far outweighs its probative or evidentiary value. But what of coincidences of both striking impact and real significance. Here even Littlewood whose mathematical background shields him from the misconceptions of the common crowd fell afoul of his own incredulity when it came to the case of two negroes, each named Will West, confined simultaneously in Leavenworth Penitentiary in 1903 and with the same Bertillon measurements\textsuperscript{74}. The case did occur and was instrumental in bringing about the use of fingerprinting for identification and the abandoning of the previously highly regarded Bertillon measurement of physical characteristics\textsuperscript{75}.

Consider the 'small world' problem\textsuperscript{76}. A person is given a document and asked to transmit it to another person he does not know in another city in another part of the United States. The procedure is to send the document to
a friend whom he knows on a first name basis and who seems likely to know the target person. The friend in turn then sends the document to one of his friends with the same instructions, and the chain continues until the document reaches the target. How many intermediate links will the chain have? Most people guess about 100. When psychologist Stanley Milgram made actual tests, he found that the links varied from two to ten and that the median was five\textsuperscript{77}.

The foregoing examples are of striking impact although of no significance - they are mere coincidences in the neutral sense of that word. There can be events not connected by coincidence but causally. Eggleston relates the Green Bicycle case\textsuperscript{78} where the defendant, aware that the police were seeking the owner of a green bicycle disposed of his bicycle and other possibly incriminating items and later denied owning them. His ownership of the bicycle and the other items was mere coincidence but his disposal of the property and his denials were not - they were a consequence of the published information that the police were seeking the owner of such a bicycle. Notwithstanding the damaging facts he was acquitted.

Tribe relates a Swedish parking case\textsuperscript{79} where a parking officer noted the position of two of the motorist's vehicle wheels before and after the vehicle had stood longer than the permitted time in a parking area. The motorist gave evidence that he had driven the car away and later returned to the same parking position which was again vacant upon
his return. The court held that although the parking officer had proved his case to the extent that there was only a $\frac{1}{12} \times \frac{1}{12} = \frac{1}{144}$ probability that the motorist was correct, it was not satisfied to the standard required to convict. Whether the court would have been prepared to convict had the officer noted the position of four wheels giving a probability of only $\frac{1}{12} \times \frac{1}{12} \times \frac{1}{12} \times \frac{1}{12} = \frac{1}{20736}$ that the motorist was correct is not clear. Tribe's discussion suggests that because the two wheels are not independent the probability of the driver's truthfulness is $\frac{1}{12}$. In fact, although the wheels are not independent, neither are they dependent in that the front steering wheels will always travel a slightly longer distance than the rear trailing wheels. It is suggested that the probability of driving the vehicle and returning with two of the wheels in their original positions is indeterminate but of an infinitesimally small value. A stopped clock will record the time correctly twice a day while a clock that loses one minute a month will only record the correct time once every 60 years.

Weaver has suggested a measure of such incidents involving coincidences as a "Surprise Index"\textsuperscript{80}. It is submitted that in attempting to determine facts, coincidences of striking impact yet lacking real significance carry excessive persuasive weight. When the occurrence of two separate events coincide the assignment of a numerical value to the coincidence is of little or no probative value unless the problem is not oversimplified and guesses are eliminated.
Notes to Chapter 3: Probabilistic Decision Making


2. Hamburg op. cit. 206-8; Weaver op. cit. 190-1; Hodges and Lehmann op. cit. 187; Reichmann op. cit. 218 ff.

3. Hamburg op. cit. 206.

4. Reichmann op. cit. 214 ff; Weaver op. cit. 189-90.

5. Ibid.


12. Ibid. 34.

13. Ibid. 313 ff.


16. Cohen does not explain the significance of this parenthetical comment.


19. \(10^{300}\) is the same as 10 followed by 299 zeros, cf. the odds in the notorious *Collins* case of about \(10^{7}\): 438 P. 2d. 33, 68 Cal. 2d. 319, 66 Cal Rptr. 497 (1968). These large numbers have been found using the Stirling approximation: Abramowitz M. and Stegun I.A., *Handbook of Mathematical Functions* (1965).


28. *Supra* ch. 2.4.


34. *Ibid*.


37. Weaver *op. cit.* 281. The Melbourne *Herald* in June, 1978 reported that the gambler had declined the club's invitation to return, all expenses paid, to mark the twenty-eighth anniversary of the event.

38. Computed by the club management to be 'about ten million to one', in fact the odds are much less: approximately one in 3,500 because the player continues to cast a "pass" by throwing any number except a 7 (in which case he loses) or the number he first threw (in which case he wins) and the game ends. Weaver *op. cit.* 213 and 248-9 describes the rules and probabilities of the game as does Kotz and Stroup *op. cit.* 72-6.

39. This would appear to be the most recent progeny of the 'bus case': *Smith v. Rapid Transit, Inc.* (1945) 317 Mass. 469, 58 NE 2d. 754.

40. Surely this constraint qualifies as "an abominably artificial assumption": n. 14 supra.

41. It is suggested that an 85 to 15 population ratio is not necessarily reflected as an 85 to 15 accident ratio. The unreality of the model is demonstrated by its failure to allocate a probability value to the possibility that such a random accident may involve two taxis of any colour. The model assumes that the plaintiff is a private motorist seeking to identify the defendant taxi company and ignores the possibility that both of the vehicles involved may be taxies belonging to one or both of the taxi companies.

42. By Bayes's Theorem:

\[
Pr(B/WG) = \frac{Pr(B) \times Pr(WG/B)}{Pr(B) \times Pr(WG/B) + Pr(G) \times Pr(WG/G)}
\]

\[
= \frac{0.85 \times 0.2}{0.85 \times 0.2 + 0.15 \times 0.8}
\]

\[
= \frac{0.27}{0.17 + 0.12} = 0.586
\]


44. \[Pr(B) = 0.8 \times 0.85 + 0.2 \times 0.15 = 0.68 + 0.03 = 0.71.\]

45. Note 42 supra.

47. *The Logic of Chance* (2nd ed. 1876) 444.

48. Adapted from Hamburg *op. cit.* 78-81; see also Hodges and Lehmann *op. cit.* 113-5; Kotz and Stroup *op. cit.* 47-8.

49. 'Roundabouts and Swings' st. 2, *Green Days and Blue Days* (1910) 19.

50. The $15 figure is termed the 'Expectation' by statisticians.

51. While it may be true that the effectiveness of the oral contraceptive pill is reputed to be 99.99%, it is a 100% failure for those women who make up the outstanding 0.01%. The ABC radio programme 'AM' on 30 August 1984 broadcast an interview with the brother of an AIDS blood transfusion victim in which he said "It only concerns ten or fifteen people but if you are one of those ten or fifteen they are a very high proportion".

52. See also Venn J., *The Logic of Chance* (2nd ed. 1876) 438.


54. Venn *op. cit.* 422.


59. Note 17 supra.


67. (1943) 52 Yale L.J. 945, 947.
68. Supra n. 66.
71. Ibid. 234.
73. Littlewood op. cit. 106.
74. Ibid.
76. Hardy, Harvie and Koestler op. cit.232.
77. Gardner (1972) op. cit. 111.
I have never seen the law of probability attempted in any case where convincing proof existed. It is only applied where no solid proof exists - where something must be fabricated from nothing before one side can prevail.


*Rowe's Rule*: the odds are five to six that the light at the end of the tunnel is the headlight of an oncoming train.

"Law and Order" (May 1978) 28(5) *Playboy* 21.

4.1 Standard of Proof

All judicial decision-making depends on an assessment of probabilities. The level of probability varies according to the circumstances of each case. It is impossible to place a mathematical value on the level of probability required in any case and in any event, the evidence introduced into such a case is usually incapable of being quantified.

4.1.1 Criminal.

On the educational level Harvard is busy collecting books, giving scholarships to persons willing to read them, and employing professors to read what the scholarship students have written. The Yale Law School is busily engaged in renting computing machines, and in hiring sociologists and psychiatrists to make a science out of law. I am informed by my spies and underpaid agents in New Haven that Yale now contemplates the employment of two astronomers and one bright Swede in order to obtain a well-rounded law faculty.

In a criminal case the standard of proof required to convict the accused has been laid down as "proof beyond a reasonable doubt". Attempts to explain or define this expression are frowned upon as more likely to lead to confusion on the part of the jury. If the jury are left in doubt concerning the accused's guilt such a jury must acquit. A quantitative measure of the standard has not been attempted except in psychological and sociological experiments. Thus it is difficult or impossible to apply quantitative methods to criminal cases. In a purported interview with a juror in a recent criminal trial the juror stated that her fellow jurors were of the opinion that a 90% probability level was too high and that a 60% probability was sufficient to satisfy the requirement of proof beyond reasonable doubt.

The legal determination of paternity involves the resolution of an issue in dispute between citizens and should thus be resolved according to the lesser standard required of a civil case wherein the defendant is not subject to the condemnation of the community and punishment by the community. However, in those jurisdictions where fornication is a crime, the proof of paternity should be to the higher criminal standard. Similarly, because of long-held moral views, adultery has been considered a quasi-criminal offence requiring the same strict standards of proof. With changing moral attitudes it is doubtful that this view will prevail although there still remains some issues that can only be resolved according to the higher standard.
Where the effect of a determination of paternity will be to confer illegitimacy upon a child there exists a common law presumption rebuttable by proof beyond a reasonable doubt that the child is legitimate. There is no analogous presumption of paternity in favour of the illegitimate child. In two Australian jurisdictions the determination of paternity that will result in the putative father's liability to maintain an illegitimate child the required standard of proof is "beyond a reasonable doubt". In the "whose baby" case in which the parentage of a child was in issue the Full Bench of the Victorian Supreme Court and the majority of the High Court on appeal held out for the highest standard of certainty notwithstanding that the case was a civil case and could not result in the stigma of illegitimacy being conferred upon an infant. The justification for this highest of standards of proof being required was the gravity of the consequences flowing from a decision that the child in issue should be taken away from the defendants and the applicants be granted custody.

Even in those Australian jurisdictions which require proof beyond reasonable doubt for the attribution of paternity it is possible for such a finding to be based on the lesser civil standard by resort to the procedure outlined in more recent legislation of the jurisdictions. Because this issue under investigation is concerned with the civil law responsibilities between the parties the criminal standard of proof will not be further discussed.
4.1.2 Civil.

Hans Solo: Never tell me the odds.

In those cases where it is required to show parentage such as to discern inheritance or the liability to maintain infants the standard of proof is the civil standard and usually described as the "preponderance of probabilities", the "balance of probabilities" or "more likely than not". In the context of the British legal system such civil cases usually involve two only adversaries and while it may be undesirable to quantify the civil standard of proof it is theoretically possible to place a numerical value of "in excess of 0.5" as the standard required. Thus, in theory at least, a value of 0.50001 will suffice although such a value is usually framed in a verbal description and not quantified.

While a third standard of proof of "clear, strong and cogent" evidence lying somewhere between the civil and criminal standards of proof appears to be recognised in the United States, it is clear that neither the English or Australian jurisdictions acknowledge more than the two standards. However, while the existence of a third standard is denied it is recognized that some civil cases require a higher standard of proof than others: where a civil case raises the question of the commission of a crime the standard of proof remains the civil standard
of proof subject only to the rule of prudence that the finder of fact should act with much care and caution before finding that a serious allegation is established\textsuperscript{22}.

In a civil case where there are more than two parties involved, each party seeking to establish their own case it may well be that the expressions "more likely than not" and "on the preponderance of probabilities" are not synonymous. For example, if a number of slips of paper are placed in a hat: four blue slips, three red and three white slips and one drawn at random it is not more likely than not that a blue slip will be drawn. Yet the preponderance of probability favours the blue slip. The difficulties of applying this game analogy is that the human affairs to be investigated in a legal hearing are rarely so simple and discretely mutually exclusive and yet it is not impossible that such a question may arise in a court case\textsuperscript{23}.

It is concluded that the issue of parentage in Australia is to be decided on the civil standard of proof. Notwithstanding such statutory expression as "reasonable satisfaction" as the standard of proof which McInerney J has described as the legislative adoption\textsuperscript{24} of the standard stated in \textit{Briginshaw v Briginshaw}\textsuperscript{25} which was "the ordinary standard of proof in civil matters"\textsuperscript{26} it should be noted that both those jurisdictions requiring proof "beyond reasonable doubt" define that standard as the requisite standard for the court to be reasonably satisfied\textsuperscript{27}. 
4.2 Corroboration

We have it from the horse's mouth of Bertrand Russell, a sound mathematician, that maths is the only science where one never knows what one is talking about, nor whether what is said is true.


He that leaveth nothing to Chance will do few things ill, but he will do very few things.

French proverb.

Although corroboration is not generally required in court cases it is required or desirable to have such corroboration in those instances where allegations are easily made and difficult to disprove. At one end it is always desirable to adduce corroborative evidence in that it will assist in establishing that which the party leading such evidence seeks to prove. The other extreme is the legal requirement in some cases where a proposition cannot be proved without corroborated evidence. There is also a common law requirement in some cases where a proposition can be proved without corroborating evidence but a jury should be warned of the dangers of doing so.

The corroboration of a witness's testimony has been the subject of many of the treatises on probability where it is sought to quantify the credit to be attached to combinations of the reports of witnesses of various degrees of trustworthiness. Cohen treats corroboration and convergence separately in that corroboration is provided by supporting oral testimony and convergence is the
consequence of the juxtaposition of items of circumstantial evidence. This distinction between corroboration and convergence is not recognized in law and will be discussed under the one head: corroboration.

Cohen describes as "the traditional, Bernoullian analysis" formula

\[ W = \frac{pq}{pq + (1-p)(1-q)} \]

where, of two witnesses A and B, p is the probability of A generally speaking the truth and q is the probability of B generally speaking the truth and W is the probability that both A and B speak the truth. Cohen proceeds to demonstrate that the formula is unsatisfactory when both p and q have values of less than 0.5 because the formula produces a lower probability for the joint veracity of both A and B whereas normal juries would assign a higher probability. In effect, the testimony of A and B tends to undermine the value of the testimony of the other witness. Eggleston shares Cohen's concern and conclusion that the traditional formula is unable to provide a proper measure of the joint veracity of the two witnesses. From this writer's view the conclusion that the joint veracity of two independent witnesses each of veracity 0.2 is only approximately 0.06 is not at all disturbing. It is only another way of saying that the joint mendacity of two independent witnesses each of mendacity 0.8 is approximately 0.94.
Cohen does not attempt to resolve or improve the formula discussed - its failing tending to support his "inductive Baconianismistic probability". Eggleston offers as an alternative a Bayesian analysis and Venn, while not accepting that the problem is amenable to a mathematical formulation, suggests the modification

\[ W = \frac{pq}{pq + (1-p')(1-q')} \]

where \( p' \) and \( q' \) represent the probability of A and B respectively speaking the truth when the event being described has not happened. It has previously been noted that the veracity of the witness may vary according to the fact he is describing in his testimony.

What is surprising is the failure of both Cohen, given his professed familiarity with Bayes' Theorem, and Eggleston, given his advocacy of utilizing Bayes' Theorem, to recognize the formula as merely the consequence of Bayes' theorem.

Given that both A and B agree in their testimony, the possible events to be considered are two in number: either A and B agree because their testimony is truthful or A and B agree because their testimony is not. The problem as defined does not admit the possibility that A and B give different testimony. Thus of the four possible events

(i) A and B both testify truthfully,
(ii) A and B both testify untruthfully,
(iii) A testifies truthfully while B testifies untruthfully, and
(iv) A testifies untruthfully while B testifies truthfully, only the first two events are relevant. The Venn diagram (figure 17) shows the sample space of all events.

Figure 17: Sample space representing the events
(i) both witnesses agreeing truthfully,
(ii) both witnesses agreeing untruthfully,
(iii) witnesses disagreeing, the first truthfully and the second untruthfully, and
(iv) witnesses disagreeing, the first untruthfully and the second truthfully.

While the Venn diagram (figure 18) shows the modified sample space subject to the condition that A and B agree in their testimony.

Figure 18: Sample space of the events where the testimony of both witnesses is in agreement
(i) both witnesses truthful, and
(ii) both witnesses untruthful.
Eggleston's supposed Bayesian analysis is founded on the unacceptable joint veracity of A and B being 0.22 when their individual veracity is 0.3 and 0.4 respectively as shown in figure 18. Eggleston postulates that, given that A and B may be inclined to tell an untruth, there may be any number of untruths available to them (he uses five in his example) and that each untruth is as equally likely as any other untruth. Substituting into the formula for Bayes' Theorem he obtains a joint veracity probability of 0.88 or in odds form 7.14 to one favouring the proposition that both A and B are truthful witnesses. In fact the joint veracity of A and B is greater than the sum of the individual veracities.

Eggleston's analysis can be illustrated by the Venn diagram (figure 19) where there are a total of thirty-six events of differing probabilities.

Figure 19: Sample space of 36 events, one of which represents both witnesses testifying truthfully, five of which represent both witnesses testifying untruthfully in a common lie, ten of which represent one witness testifying truthfully and the other testifying falsely by describing one of five lies and twenty events representing both witnesses testifying falsely although describing different lies.
The events illustrated in figure 19 are

(i) that witnesses A and B both testify truthfully;
(ii) that A speaks truthfully and B tells lie \( L_1 \);
(iii) that A speaks truthfully and B tells lie \( L_2 \) 
    \( \text{et cetera} \)

(vii) that A tells lie \( L_1 \) and B speaks truthfully;
(viii) that both A and B tell lie \( L_1 \);
(ix) that A tells lie \( L_1 \) and B tells lie \( L_2 \);
(x) that A tells lie \( L_1 \) and B tells lie \( L_3 \) 
    \( \text{et cetera} \)

(xiii) that A tells lie \( L_2 \) and B speaks truthfully;
(xiv) that A tells lie \( L_2 \) and B tells lie \( L_1 \);
(xv) that both A and B tell lie \( L_2 \);
(xvi) that A tells lie \( L_2 \) and B tells lie \( L_3 \) 
    \( \text{et cetera} \)

(xix) that A tells lie \( L_3 \) and B speaks truthfully;
(xx) that A tells lie \( L_3 \) and B tells lie \( L_1 \);
(xxi) that A tells lie \( L_3 \) and B tells lie \( L_2 \);
(xxii) that both A and B tell lie \( L_3 \) 
    \( \text{et cetera} \)

(xxv) that A tells lie \( L_4 \) and B speaks truthfully;
(xxvi) that A tells lie \( L_4 \) and B tells lie \( L_1 \) 
    \( \text{et cetera} \)

(xxxvi) that both A and B tell lie \( L_5 \).
Figure 20 illustrates Eggleston's Bayesian analysis.

Figure 20: Eggleston's Bayesian analysis with two event spaces:
(i) that both witnesses testify truthfully, and
(ii) that both witnesses testify untruthfully by describing a particular common lie out of five such lies available to the witnesses.

Eggleston has incorrectly imposed the condition that if both A and B are untruthful, they only have one of five possible lies available to them. Bayes' Theorem can be extended to more than two disjoint prior probability values and it follows that the correct analysis is that of A and B both telling the truth given that their testimonies agree. Numerically the probability value is 0.59 or in odds form 1.43 to one favouring A and B both telling the truth. This is illustrated by the following Venn diagram (figure 21) where all the possible events of A and B agreeing are shown, one of which represents the truth and five events representing the same lie as told by both A and B.
Figure 21: Bayesian analysis of six event spaces:
(i) that both witnesses testify truthfully;
(ii) that both witnesses testify untruthfully with a common lie \( L_1 \);
(iii) both witnesses testify untruthfully \( L_2 \);
(iv) both witnesses testify untruthfully \( L_3 \);
(v) both witnesses falsely testify to \( L_4 \);
(vi) both witnesses falsely testify to \( L_5 \);

This analysis, which is submitted to be the only correct analysis, permits the joint veracity of A and B to take a higher value than the veracity of either alone but not to the high degree attained by Eggleston. Eggleston's analysis can scarcely be described as "a more rigorous analysis, using Bayes' theorem". 44

The actual choice of the number of available lies available to the witnesses will affect their joint credibility. Given the individual credibility of A and B as 0.3 and 0.4 respectively, the joint credibility of both witnesses agreeing is 0.74 given that they have ten lies of equal probability available and 0.84 when they
have twenty lies available to them. Eggleston neglects to describe the method by which the number of possible lies can be enumerated. A further extension of the "Bernoullian analysis" is

\[ w = \frac{pqr}{pqr + (1-p)(1-q)(1-r)} \]

where there are three witnesses who agree in their testimony and each has an individual veracity value of p, q and r respectively. It is suggested that an appropriate graphical representation would be that of a three-dimensional cube.

Cohen then discusses a principle attributed to Ekelof and of recent origin. This has already been fully discussed by Professor Williams and replied to in an acrimonious response by Cohen. The discussion centres on the formula

\[ w = p + q - (p \times q) \]

which bears an uncanny (albeit unrecognized) resemblance to the arithmetic rule for the probability of the union of two independent events. Cohen's criticism is directed towards the failure of this principle to comply with the principle of complementation. Cohen's illustration is based on the testimony of two witnesses, the first who testifies that the offender was male on the evidence that the offender had long hair and the second witness, who is a supporter of the women's liberation movement - both witnesses are credited with a veracity value of 0.25 and according to the "Ekelof principle", the combined veracity of their joint testimonies is 0.44. This can be illustrated
by the Venn diagram (figure 22) showing 0.25 credibility for the first witness's testimony, 0.25 credibility for the second, and 1/16th (or = 0.06) for the co-incidence of their testimonies. Thus, there is 0.19 probability that the criminal was male because he had long hair, 0.19 probability that the criminal was male because a women's liberation supporter says so and 0.06 probability that the criminal was male based on both these previous factors assuming that these probability values are uniformly distributed over the sample space.

Contrary to Cohen's conclusion, the probability that the criminal was a female is 0.56 (and not 0.94) and thus the principle of complementation is not violated. His conclusions and assertions regarding the error of the "Ekelof principle" and his explanation of the failure leave much to be desired. Cohen suggests that the problem is bound up with the probabilities involved (0.25) being less than the prior probability suggested by Cohen to be of value 0.5. Cohen neglects to provide the source of
the prior probability and its quantification at 0.5. The actual source of error plaintively sought by Cohen without success is his fallacious equating of
\[ w = p - H q - (p \times q) \]
where
\[ \bar{w} = 1 - w = 1 - (p + q - (p \times q)) \]
\[ = 1 - p - q + (p \times q) \]
\[ = \bar{p} - 1 + \bar{q} + (p \times q) \]
\[ = \bar{p} + \bar{q} - (1 - (p \times q)) \]
\[ = \bar{p} + \bar{q} - (\bar{p} \times \bar{q}) \]
Cohen has failed to recognize that \( \bar{p} \times \bar{q} \) is not equal to \( \bar{p} \times q \) and consequently the probability that the offender is a female, notwithstanding the testimony of the two witnesses, is 0.56 and not 0.94 as found by Cohen.

It is concluded that with regard to the mathematical quantification of corroboration there are at least two impediments:

(i) As human affairs do not lend themselves to quantification, the coincidence of human affairs to do so is correspondingly reduced. It is, at best, only theoretically possible to quantify given that our knowledge is incomplete.
(ii) The application of such quantification is seriously hampered by the poor understanding of the mathematics involved.

Further, the testimony of a witness is not a simple "Yes" or "No" to a single question. It is possible to have witnesses agreeing on some points, actively disagreeing on others and perhaps passively disagreeing on further points in that their observations neither permit them to agree or disagree with the observations of other witnesses.

It is suggested that although Cohen's conclusions with regard to the utility of assigning mathematical values of probability may well possess merit, his reasoning renders him a poor advocate for his thesis.
Notes to Chapter 4 : Legal Determination of Facts in Issue


15. *Maintenance Ordinance 1968* (A.C.T.) s. 31(3); *Maintenance Act* (N.T.) s. 31(3).


17. *Status of Children Act* 1978 (N.T.) Sections 9-12 and 16; *Maintenance Act* (N.T.) s. 31(4). Although the *Maintenance Act* (N.T.) was based upon the *Maintenance Ordinance 1968* (A.C.T.) the A.C.T. Ordinance has not been amended to the same extent as the N.T. Act which has had s.31(4) and s.101A added to it. The A.C.T. Ordinance accords recognition and will give effect to determinations made in other jurisdictions.


21. Ibid.

22. Ibid. 111-2; Eggleston (1983) *op. cit.* 139.

23. cf. the discussion ch. 1, note 69 supra.


25. (1938) 60 CLR 336.

26. Ibid. 347 per Latham CJ.

27. *Maintenance Ordinance 1968* (A.C.T.) s. 31(3); *Maintenance Act* (N.T.) s. 31(3), (4) and s. 101A.


30. Ibid. 190-201.


36. Cohen *op. cit.* ch. 10.


38. Venn *op. cit.* 446.


42. *Ibid.* 206-7. See also Venn *op. cit.* 452 ff.


48. See chapter 2.3 note 72 supra.


5. LEGAL DETERMINATION IN CASES OF DISPUTED PATERNITY

Former criminal-court judge Louis R. Rosenthal recently admitted that he liked to let the gamble fit the crime. In order to speed settlement of charges against the three-card-monte dealers who run games on Brooklyn sidewalks, he would offer defendants a chance to plead guilty and pay a fine or plead innocent and take their chances on their own game.

Rosenthal would set out three slips of paper, face down. Each had a different finding: 30 days in jail, a $500 fine, charge dismissed.

'I'm going to mix up these papers, and he's going to pick one,' he would inform the defendant's attorney. 'They would always plead guilty - they were afraid of the 30 days.' Rosenthal claimed that the state saved at least $250 each time a trial wasn't prolonged.

He revealed his high-stakes brand of jurisprudence while testifying at the misconduct hearing of fellow judge Alan I. Friess - who was accused of deciding the length of a jail sentence by flipping a coin.

"Brooklyn Roulette" (July 1983) 30(7) Playboy 22.

Even in Toronto, which has a great safety record, if it's you that is involved, all the statistics go out the window.

John Lattanzio, Letter to the writer (October 1985).

5.1 Statutory Provisions

There are a number of statutory provisions for the determination of parentage. They are similar and in the main recognise blood testing and require corroboration of a mother's sworn evidence.
5.1.1 The Family Law Act 1975 (Commonwealth)

This Act provides for the resolution of the question of paternity of a child and empowers the Family Court to direct testing of the child, the mother and any relevant person. Given the deeming provisions of the Act wherein certain children are deemed to be children of a marriage it is difficult to envisage the need for resolving questions of paternity although recently the High Court has held that the Commonwealth Parliament does not possess the legislative power to deem children to be children of a marriage where there is no connection in fact between the child and the marriage. Thus the provisions for medical testing to resolve questions of paternity may assume greater importance with the progressive striking down of the deeming provisions.

The Act is not restricted to blood testing in that it encompasses "prescribed medical procedures" although the procedures envisaged by the Attorney-General's Department are blood tests. The prescription by regulations has not at the time of writing been made.

The Act purports to empower the Family Court to direct a person to submit to prescribed medical procedures including a person who is a relevant person in relation to the child if, in the opinion of the court, the information that could be obtained if the prescribed medical procedure were performed on that person might assist the preparation of a report concerning the paternity of the child.
It is suggested that the legislative intent is to include within the description of a relevant person an alleged or putative father. In fact the legislation is wider and may well include relatives of an alleged father such as siblings or parents. This is because the single most powerful blood test, HLA (for Human leucocyte antigen) testing, is incapable of resolving the genotype or haplotype of any given individual without further testing of the individual's blood relatives.

Whereas HLA testing can detect the presence in an individual of a number of genetic markers or antigens present on that person's sixth pair of chromosomes, the test cannot determine which antigens are located on each chromosome of the pair. Thus, where a sixth chromosome carries one antigen at locus A and another at locus B and the second sixth chromosome of the pair carries another antigen at locus A and another at locus B the test can only detect the presence of the four antigens without reference to which one of the pair carries which two antigens - the test can only determine the tested person's phenotype. If a locus A antigen and a locus B antigen can be allocated to a particular sixth chromosome of the pair then that chromosome's haplotype is determined and if both haplotypes (one for each sixth chromosome) are known then the genotype is also known. Consider a person of known genotype HLA-A11/B27 and HLA-A2/B7 which denotes that of that person's pair of sixth chromosomes, one of the chromosomes carries the antigen A11 at the A locus and the antigen B27 at the B locus while the other chromosome
(of the pair) carries antigen A2 at the A locus and B7 at the B locus. The person of known genotype HLA-A11/B27 and HLA-A2/B7 is known to possess the two haplotypes (one for each chromosome), one being HLA-A11/B27 and the other haplotype being HLA-A2/B7. The person of known genotype HLA-A11/B27 and HLA-A2/B7 has a phenotype of HLA-A2, A1, B7, B27 and the phenotype merely describes the presence of the four antigens without signifying any relationship between the four antigens.

Without testing the individual's relatives only the phenotype can be determined\(^1\). This restraint can be illustrated by an actual case\(^2\) where the mother was found to have the phenotype HLA-A1, A3, B7, Bw35 and the child's phenotype HLA-A1, A1, B7, B27. Unless the two phenotypes are compared it is not possible to determine the haplotypes or the genotypes of the individuals. By comparing the two phenotypes in this case the antigens A1 and B7 are common to both the mother and child. Thus it can be deduced that the mother's genotype is HLA-A1/B7 and HLA-A3/Bw35 and the maternal haplotype inherited by the child is HLA-A1/B7. Thus the child's other (paternally inherited) haplotype is HLA-A11/B27. In this case the putative father's phenotype was HLA-A2, A1, B7, B27. Unless further testing is carried out on the putative father's relatives it is impossible to determine the putative father's genotype or haplotype. In this case the putative father may have the genotype HLA-A2/B7 and HLA-A11/B27 which means that he cannot be excluded as the father of the child because he is capable of contributing the required paternal haplotype HLA-A11/B27. On the other
hand, the putative father may have the genotype HLA-A2/B27 and HLA-A11/B7 which would exclude the putative father because neither of his two haplotypes (HLA-A2/B27 and HLA-A11/B7) correspond to the child's inherited paternal haplotype (HLA-A11/B27). It is respectfully submitted that it is not correct to conclude that "the putative father was found to have the paternal haplotype required" in the absence of further testing of relatives of the putative father. For example, if a parent of the putative father was also tested then the putative father's genotype (and haplotypes) could be determined in the same manner as the comparison of the mother's and child's respective phenotypes provided their respective genotype (and haplotypes) provided.

The problem inherent in the case described is compounded by the frequency statistic that the genotype HLA-A2/B27 and HLA-A11/B7 is approximately four times more common than the genotype HLA-A2/B7 and HLA-A11/B27. Thus, it could be argued that while the putative father's phenotype does not exclude him as the father of the child, his likely genotype does exclude him as the father. Of course, the analysis of the putative father's genotype could be found by testing his relatives.

It is doubted that the Family Law Court possesses the power to direct a parent or sibling of a putative father to submit to a prescribed medical procedure notwithstanding that the Family Law Act purports to confer such a power. Should it be held that the Family Law Court possesses the power to direct such a parent or sibling of a putative father to submit to such testing the power may be valueless.
in that the Act expressly provides that a failure to comply with such a direction does not constitute contempt of court. Although the court is empowered to draw an inference from a person's failure to comply with such a direction it is difficult to envisage what inference, favourable or otherwise to the putative father, could be drawn from the failure of a relative of the putative father to comply with such a direction. It is of course a different matter if the putative father himself were to refuse to comply with the direction that he submit to the testing.

Although it is technically possible to test deceased persons the legislation has (perhaps wisely) refrained from conferring this power on the court.

Besides the statutory power to direct submission to testing discussed above, the common law power to order such testing has only rarely been considered by the courts. In 1949 Barry J at first instance held that the Supreme Court of Victoria possessed the power to order blood testing although he declined to exercise that power. The basis of his judgment was the public interest requirement of the due administration of justice. On appeal, no view was expressed on this point by the Full court, nor by the High Court of Australia when the case came before it on appeal. Although the case was later cited by counsel in argument before the House of Lords, the judgments of the Law Lords did not seek to distinguish the opinion of Barry J when that court held there was no power at common law to order the submission to a medical blood test.
There is no requirement in the legislation for corroborative evidence and it is submitted that corroboration is desirable merely for the parties to present their strongest case. The prerequisite for the legislation to be invoked is that the question of paternity of a child is in issue. Notwithstanding that the Family Law Act is not concerned with fault, it is suggested that blood testing will not be directed when paternity is not in issue and one party wishes to adduce evidence on another issue.

5.1.2 Victoria

This state has no statutory provisions for blood testing. Power to order blood tests, if it exists, resides in the common law. Barry J has held that such a power exists at common law although there are contrary authorities. The commonly adopted procedure is for the parties, if the parties desire such testing, to seek a consent order wherein the court requests the assistance of a named qualified immunologist or his nominee to conduct such tests.

The Maintenance Act 1965 provides that in affiliation proceedings against a putative father it is not necessary for the mother of the child to give sworn evidence, but if she does give such evidence the court cannot accept such evidence unless it is corroborated in a material
particular\textsuperscript{28} except in those instances where the putative father declines to give sworn evidence denying paternity or declines to attend the hearing after having been duly served personally with the summons to attend. In such exceptional cases the court has a discretion to accept the uncorroborated evidence of the mother\textsuperscript{29}. The Act further provides that if the mother was a common prostitute or had had sexual intercourse with men other than the defendant at or about the time of conception the defendant shall not be taken to be the father of the child\textsuperscript{30}. Thus it would appear that intercourse with men other than the defendant requires more than one other man\textsuperscript{31} although evidence that the mother had had intercourse with only one other man at about the relevant time may prevent the court being satisfied that the defendant was the father\textsuperscript{32}. It would also appear that this provision precludes a court from taking the defendant to be the father no matter how overwhelming the evidence is that the defendant is the father\textsuperscript{33}. This statutory provision may well require amendment given present scientific methods and the promise of further advances\textsuperscript{34}.

In order to satisfy the requirements for corroboration\textsuperscript{35} in affiliation cases the principles were laid down by Sholl J in \textit{Popovic v Derks}\textsuperscript{36}. Sackville and Lanteri\textsuperscript{37} suggest that, following \textit{Kenny v Hornberg (No. 2)}\textsuperscript{38} the courts are prepared to accept flimsy evidence in such affiliation cases and that "nocturnal adventurers [should] take special care to use the name of a vulnerable acquaintance when booking motel rooms"\textsuperscript{39}.  

5.4.3 Other Australian Jurisdictions

With the exceptions of the Australian Capital Territory, Western Australia and Victoria all the Australian States and the Northern Territory have recognised the evidentiary value of blood testing in questions of parentage\(^{40}\). The law in the Australian Capital Territory and Western Australia\(^{41}\) is similar to that of Victoria although neither jurisdiction has an equivalent to that of Victoria where the court cannot make an order against a defendant where the mother is a common prostitute or has engaged in sexual intercourse with other men besides the defendant at the time of conception\(^{42}\).

NSW was the first jurisdiction to recognise the evidentiary value of blood testing although the legislation was not brought into operation\(^{43}\).

The Queensland legislation empowers the Court or a judge to direct persons named in the Court's order to submit themselves for medical testing\(^{44}\). NSW permits such orders to be made concerning any party to the proceedings including the mother, the child and any person alleged to be the father\(^{45}\). SA provides for the defendant to apply to the court for a direction that the mother, child and defendant submit to blood testing\(^{46}\) while Tasmania only empowers a judge in chambers to order the testing of the child and any person alleged to be the parent of the child\(^{47}\). It is suggested that the SA legislation is
deficient in that a mother, the usual complainant, cannot apply for orders directing testing while the Tasmanian legislation may not be wide enough to empower the testing of the mother. It is essential that the mother be tested and it is desirable that relatives of the alleged father be tested although only the Queensland legislation appears to confer such a power to direct the testing of persons named in the court's order. It is suggested that the respective Parliaments have considered only the immediate persons involved being the mother, child and alleged parent.

All of the states have legislated requiring the mother's testimony to be corroborated in a material particular before a court can make an order against an alleged father.

The evidentiary value of blood testing is recognised by the various enactments such that conclusions regarding the absolute exclusion of an alleged father and a conclusion regarding the significance of the testing when the alleged father is not excluded by the testing as the father of the child is admissible evidence.

Similarly the courts are empowered to draw inferences from a person's failure to comply with an order directing submission to testing.
5.1.4 Other Jurisdictions

England\textsuperscript{53}, New Zealand\textsuperscript{54}, Canada\textsuperscript{55} and the United States\textsuperscript{56} have also legislated to empower the testing of the mother, child and the alleged father. The provisions are similar to those of the Australian jurisdictions with regard to the power to direct such testing, the persons who can be directed to submit to the testing, the value of the testing for both exclusion and inclusion results and the evidentiary provisions where a person fails to comply with an order directing testing. Corroboration of the mother's testimony is not required by the statutory provisions in England and the United States although such corroboration is obviously desirable to strengthen a party's case\textsuperscript{57}.

5.2 The Role of Probability in Parentage Testing

5.2.1 Principles of Genetic Inheritance

Yesterday Random Man died and since he didn't have a piece of the rock, Saint Peter met him and said, 'Now, you have to pass a test here before you can be admitted to Heaven.' He says, 'You see out there all the men who ever lived, and over here are all the women who ever lived. You notice they are all nude. But the test here is that if you can go out and find Adam,' he said, 'then we will admit you into Heaven.'

And the fellow said, 'That's no problem.' So he went out and about half an hour later he came back and said, 'Here's Adam.' Saint Peter said, 'That's amazing.' He said 'People have taken thousands of years,' and he said, 'You did it in half an hour.'
He said, 'It was simple.' Saint Peter said 'What do you mean?' He said, 'I just looked for somebody without a navel.'


The field of blood cell genetic typing has not risen because somebody wants to help lawyers solve disputed paternity cases. It has its origins in the study of blood transfusion reactions, tissue transplant rejection, disease correlation and anthropology studies.

Nothwithstanding doubts cast on Mendel's scientific integrity his conclusions - the Mendelian law of inheritance - are regarded as almost beyond reproach. Geneticists would be astounded to find out that it is seriously defective.

The cell theory, first proposed by Achleiden and Schwann in 1839, provides for the "growth" or creation of cells by a process of cell division wherein a cell grows and divides into two identical daughter cells; this process is called mitosis. An exception to the mitotic process is that of meiosis where, in the formation of the sex cells (the sperm and the egg), the two cells each contain one only of each pair of the twenty-two pairs of autosomal chromosomes with the female egg containing as its twenty-third chromosome the X chromosome and the male sperm containing as its twenty-third chromosome either an X or a Y chromosome. Union of the sperm and the egg during
fertilization results in a zygote (a fertilized egg) which now contains twenty-two pairs of autosomal chromosomes, one of each pair being contributed by the (unfertilized) female egg and the other of each pair being contributed by the (fertilizing) male sperm. Additionally the zygote will have as its 45th and 46th chromosomes a pair of X chromosomes (if the fertilizing sperm contained an X chromosome) or an X and a Y chromosome (if the fertilizing sperm contained a Y chromosome). A zygote containing a pair of X chromosomes will develop into a female offspring and a zygote containing one each X and Y chromosomes will develop into a male offspring of the parents. Thus, while each parent has 22 pairs of autosomal chromosomes and a 23rd "pair" of chromosomes (XX for the mother and XY for the father) the offspring will inherit one of each pair from each parent. In the case of chromosome 9 (for example), the father has two such chromosomes as has the mother. The offspring of these parents will also have two chromosomes 9, one inherited from the father and the other from the mother.

Although the definition of a gene is not precise, it is now generally recognized that genes are on or are part of a chromosome, if in fact a gene can be described as the smallest indivisible unit of inheritance. It is estimated that each chromosome carries approximately 200 different genes - the number is imprecise because not all genes have yet been identified - with each gene situated at a particular locus of the chromosome: much like beads on a string.
By way of illustration the HLA system will be used. The genes are carried on chromosome 6 at four different locations: the A, B, C and D loci with 21 identified possible genes at the A locus, 39 at B, 8 at C and 14 at D.

Any particular individual possesses two chromosomes 6 in each of his many nucleated cells. On one of his chromosomes 6 there will be one A locus gene, one B locus gene, one C locus gene and one D locus gene. On his other chromosome 6 he will have another four genes respectively at loci A, B, C and D. Usually each of the paired loci genes will be different and he is described as heterozygous. However it is possible for the same gene to be inherited from each of his parents and he is described as homozygous in that locus. An example is provided by the parents with the genotypes:

Mother: HLA-A1/B7 and A3/Bw36
Father: HLA-A2/B27 and A11/B7

It is to be noted that HLA genetic typing is usually confined to testing for the A and B loci only given that these loci provide the most information and the expense and technical difficulties associated with testing the C and D loci genes. If necessary, testing for the C and D loci genes can be conducted if the results of the A and B loci tests are insufficiently conclusive. The usefulness of the C and D loci is limited because of the high proportion (~50%) of currently undetectable antigens at these loci.
The suggested parents can at most have four HLA genetically different offspring. With five children two at least must possess the identical HLA-A and B genotype. The four possible genotypes are:

- HLA - A1/B7 and A2/B27,
- A1/B7 and A11/B7,
- A3/Bw36 and A2/B27,

Of the four possible children of different genotypes the first, third and fourth are all heterozygous in both the A and B loci genes. The second child is heterozygous in the A locus genes and homozygous in the B locus gene in that that child has inherited two B7 genes, one from each parent.

5.2.2 Detection of Genes

The chromosomes of a cell are present within the cell nucleus. Although red blood cells do not possess a nucleus and thus cannot have any chromosomes such cells, before they became red blood cells, did possess a nucleus and the usual complement of chromosomes. All other human body cells are nucleated (possess a nucleus). The chromosomes, which are made up of strands of DNA (deoxyribonucleic acid), produce proteins and enzymes in blood serum and lysates in blood cells through the medium of RNA (ribonucleic acid) which translates the genetic information carried in DNA into the proteins and enzymes. These proteins
and enzymes are expressed on the cell surface as antigens which remain after the red blood cell loses its nucleus. Thus the genetic information of a cell can be determined for all cells, both nucleated cells and those which have lost their nucleus.

Antigens are detected by serologic tests and electrophoresis by specific anti-sera or antibodies. The detection of the possible human origin of a blood-stain is determined in a similar manner. If a rabbit is injected with a small amount of human blood, the rabbit's immune system will produce anti-human sera. If the rabbit is again injected with human blood the reaction between the second injection of human blood and the rabbit's anti-human sera will be adverse, possibly killing the rabbit. Blood from such previously injected rabbits and containing anti-human sera can then be used as an anti-sera test against a blood-stain of unknown origin. If there is a visible reaction between the bloodstain and the anti-sera the forensic scientist is able to conclude that the bloodstain is of human origin. The adverse reaction observed when a previously injected rabbit with anti-human antibodies is again injected with human blood is a similar phenomenon to that observed when an Rh-negative mother carrying an Rh-positive foetus is immunized at the time of the birth of the child. The mother's immune system will produce anti-Rh-positive antibodies which are capable of passing through the placenta and can thus threaten the life of any second Rh-positive foetus or even the life
of the mother if there is a transplacental haemorrhage during the second pregnancy\textsuperscript{85}. It was this phenomenon that led Landsteiner and Weiner to discover the second known blood group (Rhesus) in 1940 and the routine immunization of those mothers at risk\textsuperscript{85}. The reaction described is the agglutination of blood cells and large solid "clumps" of red cells precipitate out of the blood fluid. More recently lymphocytotoxicity tests have been introduced, a more reliable and reproducible technique wherein the anti-bodies of a specific anti-sera will kill those cells which are complementary to the anti-sera\textsuperscript{86}. The reaction between an antigen and its complementary anti-body is a consequence of their respective molecular shapes\textsuperscript{87}. If the antigen and an anti-body are not complementary they will not attach themselves to each other and react.

Thus it can be seen that genes are detected indirectly by way of detecting the presence (or absence) of a particular antigen on the cell surface by noting any reaction between the cells and a specific anti-sera. The problems of such indirect detection is that an as yet undiscovered antigen will not react with a known specific anti-sera and the reaction between antigens and their complementary anti-sera will not disclose the number of genes - the antigen and anti-sera reaction is unable to distinguish between a cell that has antigens on its surface because a chromosome in the cell contains a particular gene and another cell that has antigens on its surface because each chromosome of an autosomal pair contains the gene, that is, anti-
sera reactions cannot distinguish between cells that contain a particular gene and a homozygous cell that contains two of those particular genes. In the instance of HLA testing for A and B locus genes, the anti-sera reactions are unable to distinguish between genes on different chromosomes and as a consequence, only the phenotype can be determined. If a sample of blood was tested and found to react positively to anti-HLA-A11, A2, B7 and B27 anti-sera, the immunologist is unable to determine the genotype or the haplotypes of each chromosome. The genotype of such a blood sample could be either HLA-A11-B27/A2-B7 or HLA-A11-B7/A2-B27. Further, when testing for HLA loci A and B, only four antigens are sought - once four are detected no further A and B locus antigens can be detected. Sometimes it occurs that less than four antigens are detected for which there are two possible explanations: first, the sample may be homozygous at a given locus, and secondly, the sample may have an antigen which is currently undetectable with the anti-sera available. Thus if a sample produced positive reactions to anti-HLA-A2, A28 and B12 anti-sera in exhaustive testing the first explanation is that the sample phenotype is HLA-A2, A28, B12, B12 and the genotype is HLA-A2-B12/A28-B12. The second explanation is that the sample phenotype is HLA-A2, A28, B12, Bx and the genotype is one of either HLA-A2-B12/A28-Bx or HLA-A2-Bx/A28-B12 where Bx represents the as yet undetectable B locus antigen. Such currently undetected antigens are rare and are treated as an individual and distinct antigen of known frequency in the population where Ax and Bx both have a frequency
of less than 2%\textsuperscript{92}. Further testing of relatives and siblings of the subject will usually resolve the question of homozygosity or the "blank" (currently undetectable) antigen and will also enable the tested phenotype to be classified according to the more informative genotype\textsuperscript{93}.

Recent research suggests it is possible to directly detect the presence of genes in individual strands of DNA within individual chromosomes\textsuperscript{94} such that the problems discussed relating to homozygosity and the classification of genotypes and haplotypes will be resolved. Further the direct detection of genes should assist in detecting the currently undetectable antigens (blanks) with the currently available anti-sera.

Extensive testing is building up a vast storage of frequency statistics for genetic markers and it is these frequency statistics that are used in estimating the probability of inclusion parentage where an exclusion cannot be obtained. Such frequency statistics are also used in disease correlation studies\textsuperscript{95} and anthropological studies\textsuperscript{96} with the possibility of distinguishing between two different Chinese localities on the basis of genetic typing alone being suggested\textsuperscript{97}.

The extensive testing and research has increased the known and detectable number of HLA genes such that between 1981\textsuperscript{98} and 1984\textsuperscript{99} the number of recognized A locus genes has been increased by 5, B locus has been increased by 6, C locus by one and D locus by 2 thus increasing the
polymorphic genetic system in humans. In testing for A and B loci alone the number of known haplotypes has been increased from 512 in 1981 to 798 in 1984. If the four HLA loci are tested the number of known haplotypes was 43,000 in 1981 and 108,000 in 1984. Similarly the number of distinct genotypes in A and B loci alone has been increased from 260,000 in 1981 to 640,000 in 1984 and the A, B, C and D loci genotypes numbered approximately 12,000,000,000 in 1984.

The above statistics relate only to the highly polymorphic HLA system. There are another 27 systems currently available wherein the three most commonly used, the ABO, the Rhesus and the MNs system, will, when combined with the HLA-A and B loci yield approximately 640,000,000 different genotypes. Thus, the usefulness of the genetic typing currently available is fast approaching (or has already reached) the "fingerprint of blood". Jeffreys et al. have suggested ten trillion trillion (100,000,000,000,000,000) individual combinations.

5.2.3 Probability in Genetic Inheritance

Mr. Edwards concluded that when he says of a quoted number 'that it is an interesting statistic' he means (on reflection) 'that is an interesting number, though I doubt its exactitude'. It would be agreeable if the next edition of the Concise Oxford Dictionary were to give the definition: 'Statistic (plural statistics) - a number of doubtful exactitude'.

Phillip Howard New Words for Old (1977) 102.
The government are very keen on amassing statistics. They collect them, add them, raise them to the nth power, take the cube root and prepare wonderful diagrams. But you must never forget that every one of these figures comes in the first instance from the village watchman, who puts down what he damn pleases.

Sir Josiah Stamp, Inland Revenue Dept. (U.K.) 1896-1917.

Where a mother, child and alleged father are genetically tested there are two principles of classes of exclusion of paternity:

(i) A gene cannot be present in a child unless it is present in one or other (or both) parents. This class of exclusion of paternity assumes that there can be no doubts regarding the maternity of the child.

(ii) A gene which must be passed on to all of the alleged father's offspring must be present in the child the subject of the parentage question.

Either of these exclusions is sufficient to exonerate the alleged father as being the father of the child.

It has long been accepted that blood testing is of limited value to the usual complainant: the mother, in that the most favourable result she can expect is a possibility that the person named by her as the father may in actual fact be the father. Conversely, such blood testing can exonerate absolutely an alleged father who
is found to be genetically incapable of being the father. This is because of the common frequency of most of the long established genetic markers, for example in the ABO group the O type red blood cells are found in excess of 40% of the whole population, A type is present in 40% of all Caucasians while the B type is present in excess of 10% of all Caucasians. Only type AB is sufficiently rare (~4% of all Caucasians) to be of value. Thus it was concluded that such testing cannot be taken as absolute proof of paternity and the utility of such testing is as an exclusory test.

With the discovery of the highly polymorphic HLA white cell system it is suggested that it is possible to place a probability value of inclusion where an alleged father is not absolutely excluded. The highly polymorphic nature of this system is demonstrated by the 800 odd possible combinations of the HLA-A/B haplotype of which the most common, HLA-A1/B8 is present in less than 7% of the population and if the C locus gene is considered the most common HLA-A/B/C haplotype is present in less than 1% of the population.

With the availability of HLA testing it is now suggested that the probability of the unexcluded putative father being the actual father can be calculated by comparing the frequency with which the paternal gene or genes occur in the general random population with the likelihood of the putative father passing on the paternal gene or genes.
Formulas for computing this probability value have been published and Terasaki suggests that "[t]he basic statistical formulas used in calculating the probability of paternity are predicated on Bayes' Theorem as applied by Essen-Moller"\textsuperscript{106}.

Maicken and Kayes commented thus on pedigree of paternity calculations based on Essen-Moller and Bayes' Theorem:

For example, one widely read article in the Journal of Family Law notes that 'the basic statistical formulas used in calculating the probability of paternity are predicated on Bayes' Theorem as applied by Essen-Moller'. Since the only citation is to Essen-Moller's untranslated 1938 paper, and because Essen-Moller unknowingly re-derived Bayes' formula in an awkward way, such explanations may leave the average attorney slightly perplexed\textsuperscript{107}.

Bayes' contribution to the theorem that bears his name has already been discussed\textsuperscript{108}. Similarly the correct choice of an a\textit{ priori} probability estimate is questioned. Supposedly, in the application of the Bayes/Essen-Moller formulation the a\textit{ priori} estimate is taken to be 0.5 thus equating the prior probability estimate of paternity with that of non-paternity. This choice is justified in that it is less than the known frequency of correct accusations of paternity in previously decided cases\textsuperscript{109} and in fact gives the accused man "the benefit of the doubt"\textsuperscript{110}. It is respectfully submitted that the results of other cases not involving the particular defendant before the court in the instant cases are legally irrelevant notwithstanding that one writer has suggested the adoption of an a\textit{ priori}
probability estimate from previously decided narcotics cases for the utilization of probability theory in other narcotics cases\(^{111}\). Notwithstanding Eggleston\(^{112}\), the Full Court of the South Australian Supreme Court has upheld\(^{113}\) the view of Williams that the evidence should focus on the defendant\(^{114}\). Whereas great effort has been expended in denying the admissibility of 'similar fact evidence' concerning the defendant before the court except in exceptional circumstances\(^{115}\), apparently similar fact evidence unconnected with the defendant before the court is acceptable.

It is further respectfully submitted that the prior probability estimate of 0.5 is no more than the likelihood of the father, possessing two alternative genetic characteristics (one on each chromosome of a pair of autosomal chromosomes), passing on the particular genetic characteristic to the offspring. There would appear to be no reason, given the process of cell division, why one of a pair of chromosomes should be more likely to be present in the sperm that fertilizes the female egg than the other chromosome. Thus, the probability that the gene in question is passed on is quantified at 0.5 presumably on the basis of the Principle of Indifference. Yet, in the case of the sex chromosomes the long term observations tend to support the view that there is a slight bias in favour of male births over female. A possible explanation may be that female zygotes are less robust than male and do not successfully undergo full term pregnancy as well as
the male. Or, that for some as yet unexplained reason
the sperm containing the Y chromosome (essential to produce
a male zygote) is more prevalent in successfully fertilizing
an egg. The allocation of a probability value of 0.5 to
one chromosome of an available two may in fact be correct
but present knowledge has not established its correctness.

However, the strongest criticism that can be levelled
at the choice of 0.5 for the a priori probability of
paternity is that it is no more than an arithmetical
confidence trick. In the town where half of the taxis
are green and the other half are blue, the reliability
of the 80% reliable witness is unaffected by the a priori
probability based on the taxi population. Similarly the
doctor who relies solely on the 80% reliable diagnostic
test to distinguish between the blue and green strains
of a disease will be no less successful in treating
his patients than the doctor who takes account of the
frequency distribution of the strains throughout the
population when the frequency distribution is equally
divided between the population. To allocate a probability
value of 0.5 to a putative father being the actual father
and a probability value of 0.5 to the putative father not
being the actual father does not advance the capability
of resolving the issue. Nor does it retard the capability
of resolving the issue.

It is noted that the proposed Regulations pursuant
to the Family Law Act accept an a priori value of 0.5.
Further, the description of the mathematical formula as being derived from Bayes/Essen-Moller is doubted in that the procedure used is a mere comparison of the alleged father contributing the genetic characteristic with the probability that a man selected at random could provide the genetic characteristic. Put simply the Paternity Index is calculated thus:

\[ \text{P.I.} = \frac{X}{Y} \]

where \( X \) is the probability of the unexcluded putative father passing on the required genetic marker present in the child but not in the mother. \( X \) usually takes the value of 0.5 except in the case where the putative father is homozygous in the required genetic characteristic in which case \( X \) takes the value 1.0. \( Y \) is the probability of a man chosen at random providing the genetic marker and is merely the frequency of the particular marker as observed in the general population\(^{119}\).

The Relative Chance of Paternity (R.C.P.)\(^{120}\), supposedly based on Bayes/Essen-Moller is

\[ \text{R.C.P.} = \frac{X}{X + Y} \]

which is no more than converting an "odds" form of probability (the Paternity Index) to a probability value (the Relative Chance of Paternity)\(^{121}\).

The simplicity of the arithmetic does not detract from the validity of the formulas although confidence is lessened
by the appeal to the authority of Bayes and Essen-Moller.

Given the ability to attribute an inclusion probability for unexcluded putative fathers it is desirable to assess the "accuracy" of the probabilistic argument.

Depending on the polymorphism of the system used it is possible to calculate a theoretical value of the probability of exclusion. This value is a measure of the ability of the probabilistic argument to correctly exclude the innocent defendant. The Probability of Exclusion using the ABO system has been computed to be $0.176^{122}$ suggesting that in 100 cases involving accusations of paternity against innocent men the ABO blood testing will exclude 18 such men as possible fathers while 82 such men will not be excluded. The Probability of Exclusion for the highly polymorphic HLA system is in excess of 0.85 suggesting that the HLA system alone is capable of correctly excluding 85 or more men out of 100 such innocent men.

The typing systems are independent and it is thus possible to compute the combined Probability of Exclusion using the ABO system with a Probability of Exclusion of 0.176 and the HLA system with a Probability of Exclusion of 0.85 is

$$0.85 + 0.176 - 0.85 \times 0.176 = 0.87$$

and using the more convenient formula for multiple independent events

$$124$$
\[ CPE = 1 - (1 - PE_1)(1 - PE_2)(1 - PE_3) \ldots (1 - PE_n) \]
gives a combined Probability of Exclusion for the five common red cell Antigens tested (ABO, MNSS, Rhesus, Duffy and Kidd) of 0.732 and if these five systems and the HLA loci A and B are tested the combined Probability of Exclusion is 0.97 which is in excess of the proposed minimum acceptable pursuant to the Family Law Act\textsuperscript{125}.

Race and Sanger\textsuperscript{126} attribute to Fisher an alternative measure of 'usefulness' of the different systems in their ability to distinguish between two random samples of blood.

On a non-numerical basis the measure of inclusion probability for a non-excluded putative father has the further advantage that it may corroborate the mother's evidence. The child provides an objective genetic description of its father and the genetic probabilities are based solely on genetic testing independent of any non-genetic evidence for or against paternity\textsuperscript{127}. Where the alleged father is not excluded using a number of systems with a high discrimination Probability of Exclusion value and a high likelihood that the alleged father is in fact the actual father the fact that the mother was able to name the alleged father prior to blood testing is significant\textsuperscript{128}. This point met with the approval of the Supreme Court of South Africa:

\begin{quote}
Then of course, the medical evidence is highly corroborative of the plaintiff. Plaintiff picked defendant as the father of her child long before any blood tests were done, and these tests show an overwhelming probability that her allegation is correct\textsuperscript{129}.
\end{quote}
It is concluded that such genetic testing is valuable, certainly for exclusion but also for inclusion where there is a non-exclusion. It is important to stress that a conclusion regarding the probability or likelihood of paternity in numerical form is evidence itself - it is not a measure of the weight of the evidence and thus cannot be used in an attempt to build up a numerical value of probability thought to represent a particular standard of proof. The real value of such genetic testing is immeasurable and is the corroboration, whether required by law or not but always desirable, of the mother's accusation where the accusation is made before the accuser has the benefit of the information relating to the accused man's genetic characteristics.
Notes to Chapter 5: Legal Determination in Cases of Disputed Paternity

1. Sections 99 and 99A.
2. S.99A(2).
3. S.5.
4. Cormick and Cormick v Salmon (1984) FLC 91-554 wherein s.5(1)(f) of the Family Law Act was held to be not a valid law and In re Cook and Maxwell, ex parte C (Judgment delivered 1 August 1985) wherein s.5(1)(e) was also held invalid.
5. S.99A(2).
6. Personal communication, Dr. Brian Tait of the Royal Melbourne Hospital, 3 September 1985.
7. April, 1986
8. S.99A(2).
9. S.99A(3).
10. Chapter 5.2.1. infra.
12. Ibid. 551.
15. S.99A(2), (3).
17. Ibid.
18. (Cont'd)


20. Ibid. 299.


23. Ibid.

24. Chapter 4.2., note 27 supra.


26. Supra notes 19-23.


28. Maintenance Act 1965 (Vic.) s.27(1).

29. Ibid.

30. Ibid s.27(2).


32. Ibid.


35. Maintenance Act 1965 (Vic.) s.27(i); Hambly and Turner op. cit. 512.


38. (1963) 37 ALJR 162.


40. Status of Children Act 1979 (Qld) s.11; Children (Equality of Status) Act 1976 (NSW) ss.19-22; Status of Children Act 1974 (Tas.) s.10; Community Welfare Act 1972 (SA) s.112; Status of Children Act 1978 (N.T.) ss.13-16.

41. Maintenance Ordinance 1968 (ACT) s.31; Family Court Act 1975-1979 (W.A.) s.55.

42. Supra n.31.

43. Child Welfare Act 1939 (NSW) s.120.

44. Status of Children Act 1978 (Qld) s.11(1).


47. Status of Children Act 1974 (Tas.) s.10(3).

48. Chapter 5.2.1 infra.

49. Supra n.14.

50. Maintenance Act 1965 (Qld) s.30; Maintenance Act 1974 (NSW) s.32; Maintenance Act 1967 (Tas.) s.35; Community Welfare Act 1972 (SA) s.140; Maintenance Act (N.T.) s.31(1); Family Court Act 1975-1979 (WA) s.55(3).

51. Status of Children Act 1978 (Qld) s.11(4)(h); Maintenance Act 1974 (NSW) s.20(1); Status of Children Act 1978 (NT) s.14(l)(c).

52. Status of Children Act 1978 (Qld) s.11(7); Maintenance Act 1974 (NSW) s.21; Community Welfare Act 1972 (SA) s.9; Status of Children Act 1978 (NT) s.15.


54. Family Proceedings Act 1980 (NZ) ss.54, 55.

55. Children of Unmarried Parents Acts : British Columbia s.15, Ontario s.19.

57. Supra chapter 4.2, note 27.


62. Eriksson et al. op. cit. passim.


64. Humphreys W.C. Anomalies and Scientific Theories (1968) 179.

65. Ibid.


70. Contreras R. and Fiers W., op. cit. 125; Watson op. cit. 702; Delaney and Garratty op. cit. 127; Race and Sanger op. cit. 5; Am. Assoc. of Blood Banks Technical Manual op. cit. 77-8; Simons M., "HLA Genetic Typing for Disputed Paternity" De facto Relationships : Legal Aspects (1981) 91, 92; Miller W.V., "HLA serotyping in cases of disputed paternity" Paternity Testing (1978) 55.

71. Watson op. cit. 491 ff.


75. Am. Assn. of Blood Banks Technical Manual op. cit. 98, 135; Terasaki P.I., "Resolution by HLA testing of 1000 Paternity Cases not excluded by ABO testing" (1977-8) 16 J. Fam. Law 543, 545-6; Watson op. cit. 10, 705; Race and Sanger op. cit. 5; Simons op. cit. 94.


80. Note 75 supra.


83. Ibid.; Delaney and Garratty op. cit. 8-9.

84. Picton B., Murder, Suicide or Accident (1971) 185-7.

85. Archer and Parker op. cit. 179-183, 250-7; Am. Assoc. of Blood Banks Technical Manual, op. cit. ch. 9; Delaney and Garratty op. cit. chs. 18, 20 and 21; Am. Assoc. of Blood Banks Transfusion with "Crossmatch-Incompatible" Blood (1975) 7-10; Race and Sanger op. cit. ch. 5; Picton op. cit. 188.

86. Archer and Parker op. cit. 216.


89. Refer discussion notes 11-4 supra.

90. Simons op. cit. 93; Archer and Parker op. cit. 219; Am. Assoc. of Blood Banks Technical Manual, op. cit. 131; Terasaki (1977-8) op. cit. 545-6.


92. Ibid. 546.


95. Supra n.61.

96. Supra n.62.


104. Picton loc. cit.

105. Terasaki (1977-8) op. cit. 546.

106. Ibid. 544.


108. Ch. 2.4 supra.


113. SGJC (SA) v Laube (1984) 37 SASR 31, 33; 1 MVR 417, 419.


116. Chapter 3.3 supra.

117. Ibid.


119. Ibid.

120. Ibid.


122. Race and Sanger op. cit. 505.

123. Ch. 2.3, note 71 supra.

124. Ibid. note 73.

125. Prescribed Medical Procedure (s.99A) and Para 5 of proposed "Prescribed Form of Report under Sub-section 99A(7)" : Letter from Attorney-General's Department 25 March 1985.

126. Race and Sanger op. cit. 507.

127. Terasaki (1977-8) op. cit. 549.


129. Van Der Harst v Viljoen [1977(1)] SALR 795, 796 per Watermeyer J.
6. CONCLUSION

Statistician: a person who diligently collects facts and figures and from them draws any number of confusions.


Positive, adj. Mistaken at the top of one's voice.


Chance is a word void of sense; nothing can exist without a cause.

Voltaire.

The army's emphasis on quantification (a disease it caught when Secretary McNamara sneezed) meant that success was defined only by what could be measured. This was partly caused by the computer craze and resulted in the application to the army of 'the commercial ethic'. This contributed to two unfortunate consequences: ignoring characteristics that could not easily be expressed in numbers, such as leadership, and emphasizing activities that could be measured, such as 'savings bond scores and the reenlistment rate'. Officers were promoted for doing well in these 'programs', while they were not reprimanded for failures in areas that the computer could not be programmed to measure, such as duty, honour, country.


It is concluded that mathematical probability can be utilized in a limited manner in the determination of factual issues in law. With regard to the exclusion of parentage, probability theory is irrelevant in that the Mendelian principles of genetic inheritance are absolute once it is shown that the offspring does not share the required genetic characteristic with the parent (usually the father).
With regard to the inclusion of parentage, probability theory provides a yardstick by which the coincidence of the characteristic of the alleged parent who has not been excluded can be compared with that of the general population. The present state of our knowledge only permits an assessment of the probability that a non-excluded parent is the parent, present methods do not permit an absolute attribution of parentage. Thus, it is submitted that such evidence can only serve as corroboration of other evidence. Without other evidence, the evidentiary value of the inclusion probability is of little weight and insufficiently relevant. This conclusion is given notwithstanding that the HLA system alone contains some 100,000 different genetic combinations thus satisfying the future "fingerprint of blood" discussed by Foote, Levy and Sander.  

Current research may permit an absolute attribution of parentage in the future. An analysis of the structure of a gene's deoxyribonucleic acid (DNA) permits identification and comparison with other genes and it has been suggested that there are as many as $10^{19}$ possible permutations and combinations - each one unique to a particular individual person and obeying the Mendelian principles of genetic inheritance. Furthermore current research promises accurate detection of homozygosity thus permitting the identification of all genes making up an individuals' genotype.
In attributing numerical probability values to an event under investigation by a court it must always be remembered that there is only one kind of statistical anomaly. Only distributions, frequencies and averages can be anomalous with respect to statistical laws. Individual events cannot. The description of an individual event simply cannot contradict a statistical generalization.

An individual event can, of course, be improbable (in the sense of having a low predicted probability, from known statistical laws). But improbability is not the same thing as anomalousness. Getting ten straight deuces at dice is highly improbable. But it is not anomalous - unless one holds the (mistaken) belief that highly improbable events never occur.

While it is suggested that present knowledge can only permit corroborative evidence of inclusion probabilities to be given, the evidentiary value of the evidence cannot be quantified although the evidence itself is in quantitative form. Thus the weight to be accorded to such evidence remains wholly within the function of the finder of fact in a legal hearing.
Notes to Chapter 6 : Conclusion


3. Personal communication Dr. Brian Tait of Royal Melbourne Hospital 3 September 1985.


5. Ibid. 207.

6. Ibid.