CHAPTER TEN

A Functional Analysis of Mathematical Tasks in China, Japan, Sweden, Australia and the USA: Voice and Agency

INTRODUCTION

In the mathematics classroom, the teacher, the student and the tasks provide the key structural elements through which the classroom’s social activity is constituted. In the analysis reported in this chapter, we restrict our consideration of task to activities that are recognizably mathematical. Marx and Walsh (1988) identified three essential elements to any consideration of the role of ‘academic tasks’: the conditions under which the tasks are set; the cognitive plans students use to accomplish tasks; and the products that students create as a result of their task-related efforts. This conception either ignores the role of teacher intentionality and mediation, or it relegates this to just another element in the social context in which the task is undertaken. Our conception of the teacher/student/task triad is much more interconnected and accords significant agency to each in the determination of the actions and outcomes that find their nexus in the social situation for which the task is the pretext.

More recently, theories of learning have viewed cognitive activity as not simply occurring in a social context, but as being constituted in and by social interaction (Hutchins, 1995; Salomon, 1993). From this perspective, the activity that arises as a consequence of a student’s completion of a task is itself a constituent element of the learning process and the artefacts (both conceptual and physical) employed in the completion of the task serve simultaneous purposes as scaffolds for cognition, repositories of distributed cognition and cognitive products. Task selection by teachers represents the initiation of an instructional process that includes task performance (collaboratively by teacher and student) and the interpretation of the consequences of this enactment (again, by teacher and student).

In Simon’s (1995) construct of the hypothetical learning trajectory (HLT), mathematical tasks are seen as central to the promotion of student learning. Baroody, Cibulskis, Lai and Li (2004) have drawn attention to the important distinction between learning trajectories and other learning sequences, such as Gagné’s learning hierarchies (e.g. Gagné & Briggs, 1974). A key aspect of this
distinction is the degree of prescription implicit in the learning sequence. Hypothetical learning trajectories maintain an emphasis on the hypothetical and incrementally adjusted nature of any posited learning trajectory for any student. Any investigation of the function of tasks in mathematics instruction/learning (viewed in this paper as a conjoint, co-constructed activity, see Clarke, 2001 and 2006a), must take into account intention, action, and interpretation (by both teacher and students) and view any hypothetical learning trajectories as subject to continual and incremental adjustment during the course of classroom task performance (Simon & Tzur, 2004).

In this chapter, we examine the function of mathematical tasks in classrooms in five countries. Utilising a three-camera method of video data generation (see Clarke, 2006b), supplemented by post-lesson video-stimulated reconstructive interviews with teacher and students, we can characterize the tasks employed in each classroom with respect to intention, action and interpretation and relate the instructional purpose that guided teacher task selection and use to student interpretation and action, and, ultimately, to the learning that post-lesson interviews encourage us to associate with each task.

Our analysis utilises the conception by Clarke and Lobato (2002) of ‘function’ as the combination of intention, action and interpretation to examine the functionality of mathematical tasks in classrooms in several countries. Of particular interest are differences in the function of mathematically similar tasks when employed by different teachers, in different classrooms, for different instructional purposes, with different students. The significance of differences between social, cultural and curricular settings, together with differences between participating classroom communities, challenges any reductionist attempts to characterize instructional tasks independent of these considerations. Of equal interest are differences in learning outcomes arising from the use of fundamentally different mathematical tasks, such as highly decontextualised or abstract tasks (in the Chinese classrooms, for example) in comparison with contextualised or so-called ‘real world’ tasks (for instance, in one Swedish classroom). In particular, student willingness and capacity to associate classroom mathematics with out-of-class activities was inconsistently associated with the explicitness of the teachers’ goal of establishing (or not establishing) such connections.

Data Construction

In any classroom video study, the choice of classroom, the number of cameras used, who is kept in view continuously and who appears only given particular circumstances, all contribute to a process that might best be characterised as ‘data construction’ or ‘data generation’ rather than ‘data reduction’ (Miles & Huberman, 2004). Every decision to zoom in for a closer shot or to pull back for a wide angle view represents a purposeful act by the researcher to selectively construct a data set optimally amenable to the type of analysis anticipated and maximally aligned with the particular research questions of interest to the researcher. The process of data construction does not stop with the video record, since which statements (or whose
FUNCTIONAL ANALYSIS OF MATHEMATICAL TASKS

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In the key element of the post-lesson student interviews, in which a picture-in-

picture video record was used as stimulus for student reconstructions of classroom 

ents, students were given control of the video replay and asked to identify and 

ment upon classroom events of personal importance. In addition, the classroom 

data (three independent video records plus the combined picture in picture 

and the post-lesson video-stimulated interview data were supplemented by 

ional data in the form of teacher questionnaires (before and after the lesson 

ue and after each individual lesson), scanned written and text material used 

or generated by the focus students or the teacher, classroom test material and a test 

of student mathematical achievement administered after the completion of 

videotaping.

This data set provided an extensive pool of mathematical tasks, documented in 

use in classrooms around the world. From the combination of data described 

above, it is possible not only to describe the mathematical tasks employed, but by
drawing on the complementary accounts provided by the teacher and the students, analysis of the function of tasks can include both intention and interpretation.

**Analytical Approach**

The key to our task selection for this paper was the identification of 'distinctive' tasks. For the purposes of this paper, 'distinctive' could mean either typical or unusual. While this may sound paradoxical, our primary purpose was to examine mathematics tasks in the classrooms of competent teachers around the world for the implications that these might have for instructional practice and theory. Given this goal, a task may be 'distinctive' because it represents something characteristic (typical) of the practices of a particular competent teacher, or a task may be distinctive because it is unusual. It is possible, of course, for a task to be both typical of a teacher's practice, and unusual in comparison with the practice of other teachers. In both cases, we have chosen tasks that the data suggest were instructionally effective.

**Results**

Each of the selected 'Distinctive tasks' is described below in terms of the Educational Context of the Task, the Social Performance of the Task, and the Intermediate and Consequent Artefacts arising from the social activity of performing the task. Our description of the Educational Context of each task includes the teacher's goals for the lesson, as described in the teacher interview data, teacher questionnaire data or when stated by the teacher in class.

We have chosen in this chapter to focus on nine distinctive tasks: one from each of the classrooms in Shanghai and Tokyo, plus one task from a classroom in Sweden, one from Australia, and one from the U.S.A. The last three tasks were chosen, in part, for the contrast they represented to the tasks employed in the LPS classrooms in China and Japan. In the tabular display of each task, the various contributions to the 'Social Performance of the Task' are displayed in chronological order down the page. Present tense is used in describing the Social Performance of the Task in order to provide a text account with a sense of immediacy comparable to that provided by the video record. The 'Interpretive Reflections' represent our interpretations of the video record of the task. The subsequent discussion examines similarities and differences across the selected tasks, particularly from the perspectives of the distribution of responsibility for the generation of new knowledge in the classroom and with respect to the distribution of voice and agency within the classroom performance of each task.

Our intention in this chapter was to represent the selected tasks in such a way as to facilitate their comparison. There are many comparative analyses that might be undertaken on the tasks as performed and as documented here, but our focus concerned the extent to which the task has served as a vehicle for the distribution of responsibility for knowledge generation. Specifically, our discussion of each task addresses the question of whether or not the teacher has accorded the students
significant agency and/or voice in the development of new mathematical (in this case, algebraic) knowledge.

**Task One: Japan School 1 – Lesson 1 (the Stairs Task)**

<table>
<thead>
<tr>
<th>Educational Context of the Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>This was the first lesson in a sequence of lessons concerned with functions, relations and patterns, where particular emphasis was placed on the special terms used in mathematics. The teacher identified her global aims for the entire lesson sequence of about sixteen lessons, as i) identifying functions and their relationships to everyday life; and ii) understanding how to solve equations using a table, graph or formal solving techniques. This particular lesson was designed by the teacher to focus on i) different variables and their relationships with one another, ii) understanding the form of a linear equation; and iii) understanding that the investigation of the nature and function of equations is of utmost importance.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Social Performance of the Task</th>
</tr>
</thead>
</table>

**Mathematical Task as Stated**

<table>
<thead>
<tr>
<th>Teacher</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher states her goal to the class, “I’d like to think about change using these figures.” The teacher asks students to work on drawing the next two figures in the sequence.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Student</th>
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</thead>
<tbody>
<tr>
<td>Jitsu immediately suggests, “the number of steps.” Nobo and Nou add “size” and “area” respectively. Taka adds “height.”</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Teacher</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher invites students to present ideas on identifying what aspect of the figures is changing.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Student</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nobo and Nou add “size” and “area” respectively. Taka adds “height.”</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Teacher</th>
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<tbody>
<tr>
<td>The teacher invites students to work in small teams to identify as many aspects as they can.</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Student</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nobo and Nou add “size” and “area” respectively. Taka adds “height.”</td>
</tr>
</tbody>
</table>
The teacher roams the classroom and speaks with individual students.

The teacher addresses the class and invites further suggestions on "what changes?"

The teacher invites students to suggest how they might go about examining the relationship between the number of steps and the circumference.

The teacher reiterates Mawa's mention of the use of a table and reminds students that mathematical expressions are also useful in examining relationships. She proceeds to draw up a table:

<table>
<thead>
<tr>
<th>Number of steps</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circumference (cm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

She asks students to complete the table and to identify a mathematical relationship. (The teacher roams around the room assisting students.)

The teacher asks Yama to go to the board and trace the circumference of the staircase with four steps.

Examine the relationship between the number of steps and the circumference.

Students working in small groups:

Niu adds "number of sides" and "number of squares."
Mika adds "circumference."
Taka says, "shape."
Nou responds with "the length of the base."
Jitsu adds "the time it takes to draw the figures."
Nobo adds "sum of the interior angle" and "the number of vertices."

Some students suggest, "graphs."
Mawa mentions "table."

Students work on the assigned task at their desks.
The teacher invites a student to fill out the table and another to write the relationship as an equation.

The teacher reviews the results in the table with the class and discusses the mathematical relationship represented by the table.

The teacher asks for other interpretations – other methods for getting the answer.

The teacher highlights this method graphically with Nobo’s help.

The teacher assigns the homework task.

<table>
<thead>
<tr>
<th>number of steps</th>
<th>circumference (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
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</tbody>
</table>

Yama assists the teacher by tracing the circumference of the figure with his finger.

Ume volunteers:

\[
y = 4x.
\]

Taka writes \( y = 4x \).

Most of the students agree that to find the circumference the number of steps is simply multiplied by four.

Nii responds that you can add four to the previous answer, so that to get the circumference for six steps, one adds four to 20.

Think about how to show that ‘multiplying by four’ works.

Examine the relationship between the number of steps and another feature of the diagrams.
Interpretive Reflection

The Stairs Task is well-known and used by mathematics teachers in many countries. The task was chosen for analysis because of its visual appeal and capacity to provide a focus for student attention. The teacher presented the problem and asked the students, "What changes when the number of steps change?" The teacher then took suggestions, and in doing so made clear the sort of responses she was expecting, and then gave the students the opportunity to work with one another on the task.

The task was presented with great care and appeared quite successful in meeting the teacher's objectives as stated in the questionnaire data. In particular, the students were able to identify a large number of differing variables. It was at this point in the lesson that the teacher narrowed the focus to the particular connection between the number of steps and the circumference and began to model a mathematical approach to defining this relationship. The teacher's focus on the students and the partnership she had formed with them in their learning was evidenced by students' contributions at the board: Nou drew steps four and five; Yama traced one of the figures with his finger to illustrate the concept of circumference; Ume completed the table, while Taka wrote the relationship in algebraic terms; and Nobo shared his various conjectures of relevant variables.

Once the relationship \( y = 4x \) was proposed, the teacher's attention turned to how to demonstrate the correctness of this equation, "Think about how to show that multiplying by four works." In her questionnaire responses, the teacher identified the goal of student recognition of the possible relationships between different variables and implemented this aim with the tasks she assigned and the questions she asked.

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Task Two: Shanghai School 2 – Lesson 8 (the Numbers task)

Educational Context

The class was learning about the system of linear equations. The students have spent previous lessons working with inequalities, the relationship between the concept of linear equations in two unknowns and their solution, the transformation of equations and solution methods involving substitution and elimination.

This was the eighth lesson in this topic and one of the teacher’s goals for this lesson was for students to learn to solve some special linear equations in three unknowns.

Social Performance of the Task

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Mathematical Task as Stated</th>
<th>Student</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher presents the class with a task for all to consider. The teacher reads out the question. The teacher invites students to set up the equations.</td>
<td>There is a three digit number. The sum of the three digits is 12. The sum of the hundreds digit and the tens digit is greater than the ones digit by 2. Three times the hundreds digit equals the sum of the tens and the</td>
<td>(Students attentive)</td>
</tr>
<tr>
<td>The teacher invites a student to give her response</td>
<td></td>
<td>Carry proposes, “x plus y plus z equals twelve.”</td>
</tr>
<tr>
<td>The teacher confirms Carry’s response is indeed correct.</td>
<td></td>
<td>Charlene states, “x plus y equals z plus two.”</td>
</tr>
</tbody>
</table>

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The teacher discusses the characteristics of this system of equations.
\[ \begin{align*}
  x + y + z &= 12 \\
  x + y &= z + 2 \\
  y + z &= 3x
\end{align*} \]
This involves identifying which equations must be satisfied ("all of them") and using the exact terminology ("system of linear equations in three unknowns").

Let the hundreds digit be \( x \), and the tens digit be \( y \) and the ones digit be \( z \).

Set up the equations according to the question.

Cy adds, "\( y + z \) equals three \( x \)."

(Students verbally responding to teacher questions in unison)

**Interpretive Reflection**

We chose this task in part because of its function in introducing students to the underlying structure of algebraic representations and in assisting them to develop appropriate mathematical language. The strategic development of technical language has emerged from other analyses as a possible characteristic of mathematics teaching in China (Clarke & Xu, 2007). The task was presented to the class and three students were able to correctly provide the individual equations to form the entire system — indeed, it appeared they were able to do so quite effortlessly. It was at this point that the teacher spent some time identifying the exact nature and definitive characteristics of a system of linear equations in three unknowns.

The students were not encouraged to actually solve the equations, algebraically or by other means. The task consisted entirely of setting up the system of equations. Students were then presented with the following system to solve, understandably easier but perhaps not as enticing as the task that prompted the set of equations they had just constructed:

\[ \begin{align*}
  x + y &= 11 \\
  y + z &= 17 \\
  x + z &= 10
\end{align*} \]

**Task Three: Sweden School 1 — Lesson 11 (the Graph Task)**

**Educational Context**

This class had been working on the topic of proportionality. This involved an introduction to: the coordinate system; the construction of a graph; direct variation and non-proportionality.

This was the sixth lesson in this topic and the teacher’s target content for this lesson included: calculating price per unit from a straight line graph; determining the equation of a
straight-line graph; gradient; and determining which equations best represented a particular graph.

<table>
<thead>
<tr>
<th>Social Performance of the Task</th>
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</thead>
<tbody>
<tr>
<td>Teacher</td>
</tr>
<tr>
<td>The teacher projects the straight-line graph on a screen at the front of the class and asks students, “what can you tell me about this?”</td>
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</table>

In response to Beata the teacher adds to the diagram:

The teacher amends the diagram in response to the students’ comments:

The teacher states that the

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<tbody>
<tr>
<td>10</td>
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<tr>
<td>20</td>
</tr>
<tr>
<td>30</td>
</tr>
<tr>
<td>40</td>
</tr>
<tr>
<td>50</td>
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</table>

<table>
<thead>
<tr>
<th>quantity</th>
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<tbody>
<tr>
<td>0</td>
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<tr>
<td>1</td>
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<td>7</td>
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</tbody>
</table>

A discussion ensues with students pointing out certain features that are absent from the diagram. These include: the labelling of the x and y axes, the origin and numerical values for the dashes along the axes.
different lines now represent apples and pears. He asks the students whether they now have new information to add.

The teacher attempts, with a ruler, to read the cost of a single fruit but declares that it is rather difficult and that if they were to consider a larger quantity of fruit they would achieve a more accurate result.

After attempts to indicate on the graph the cost of 5 pieces of fruit the teacher summarises their findings so far:

- one is more expensive than the other
- one is cheaper than the other
- it has something to do with how the graph slopes

(The task is momentarily abandoned as the subsequent discussion involves the problem $90 \times 90$ and the different ways of calculating this quantity)

The teacher asks for a cost equation that summarises the information in the graph.

The teacher highlights the meaning of the $x$ in the equation $K = 15x$. They then determine that the other cost equation is $K = 10x$.

The teacher continues the conversation by asking the students to determine where

The students respond that it is now possible to find the cost of a single piece of fruit. Alva and Viktoria explain how to read the cost of a single fruit.

Annette suggests that it is, “the quantity times 15.”
$K = 2x$ would appear on the given graph.
The teacher continues by drawing up a table:

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>0</td>
<td>2</td>
<td>10</td>
</tr>
</tbody>
</table>

and with the use of these points determines the graph of $K = 2x$.

**Interpretive Reflection**

The Graph Task was chosen for its strategy of ‘creating’ a real-world context for an abstract mathematical representation. The students became responsible for giving meaning to two lines on a graph and subsequently for determining the relationship in algebraic terms by identifying the value of the gradient. The problem was introduced quite conversationally and this relatively informal presentation reflects the classroom’s social interactions. Students were encouraged to make observations about the two lines and the teacher responded to those comments that were of a mathematical nature. The teacher’s selective acknowledgement of particular responses established him as the mathematical authority in an otherwise very casual classroom environment.

The character of this classroom and of the mathematical activity appeared to be dependent upon participants being willing to make verbal contributions. The teacher’s strategy to maximise student interest and engagement appeared to be based on the conversion of an abstract mathematical situation into one that had a real-world context.

**Task Four: Shanghai School 3 – Lesson 7 (the Train Task)**

**Educational Context**

This topic was intended to promote learning about systems of linear equations in two unknowns. The students have spent previous lessons discussing the concept of linear equations in two unknowns, the rectangular coordinate plane and the graph of a linear equation, determining whether a given system satisfies the criteria to be classified as a system of linear equations in two unknowns and determining whether a particular ordered pair is a solution.

This was the seventh lesson in this topic and one of the teacher’s goals for this lesson was to demonstrate how to solve a system of linear equations in two unknowns by elimination (using addition and subtraction).

**Social Performance of the Task**

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Mathematical Task as Stated</th>
<th>Student</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher presents the class with a task for all to consider. The teacher reads out the problem and invites students to calculate an answer mentally.</td>
<td>Siu Ming’s family intends to travel to Beijing by train</td>
<td>(Students attentive)</td>
</tr>
</tbody>
</table>
The teacher asks students to explain how to calculate the cost of the student ticket.

The teacher reiterates Dora’s explanation and invites students to explain how one would calculate the cost of an adult ticket.

The teacher highlights that this solution has been achieved by inspection. He then requests students to use their recent knowledge to form appropriate equations.

The teacher demonstration of the solution to this task involves: Confirming that the system of equations the students are working with is

\[
\begin{align*}
3x + y &= 560 \\
3x + 2y &= 640
\end{align*}
\]

Subtracting the first equation from the second gives \( y = 80 \).

Substituting \( y = 80 \) into the first equation gives \( 3x + 80 = 560 \), thus \( x = 160 \).

Interpretive Reflection
The Train Task was chosen because of its integrated use of different representations. Students initially solved the word problem by inspection. Dora succinctly explained her thinking in identifying the cost of the student ticket and Eva was able to continue the calculation to find the cost of the adult ticket. The task was then represented as a pair of simultaneous equations that were then solved by the teacher, with the appropriate algebraic techniques.
By removing the burden of finding a solution early on, the teacher was able to focus his students' attention on the algebraic representation and the technique of elimination and, perhaps, also reassure students, at the same time, of the legitimacy of abstract algebraic manipulation.

**Task Five: Japan School 2 – Lesson 1 (the Equivalent Deformation task)**

**Educational Context**
This classroom was studying the topic of geometric congruence and similarity. The first three lessons in this topic sequence focused on the skill of "equivalent deformation."

The task of this lesson, the first lesson, involved learning and understanding the technique of converting a parallelogram to a triangle while keeping the area constant.

The second and third lessons extended and consolidated this skill with a more difficult example – in this additional case the class converted one triangle to another triangle, again without changing the value of the area.

### Social Performance of the Task

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Mathematical Task as Stated</th>
<th>Student</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher identifies the theme for the day as 'equivalent deformation.' Some time is spent discussing the meaning of the phrase – it is concluded that students will learn how to change the shape of a given diagram keeping the area constant. He then invites students to solve the task. A few hints are given, including: drawing a quadrilateral, reminding students to keep the area constant; identifying the problem as a construction problem thereby asking students to use a ruler; reminding students to use parallel lines (The teacher offers individual assistance to students.)  The teacher demonstrates the solution to the task: the quadrilateral is divided in two with a diagonal line.</td>
<td>Consider parallelogram ABCD  Change the quadrilateral to a triangle keeping the area constant.</td>
<td>(Students are solving the problem in their seats.)</td>
</tr>
</tbody>
</table>
The Equivalent Deformation task was chosen for its highly visual nature and because it allowed students to engage with a task that required diagrammatic manipulation, which was not a common feature of the tasks studied. We also considered it an unusual task and a challenging one; certainly this type of task was not commonly found in the classrooms we have studied.

The presentation of the task involved defining the term 'equivalent deformation' and students were then given various hints on how to perform the task. Much of the time dedicated to this task was spent by the teacher offering individual advice to students at their desks. This advice commonly took the form of encouraging them to talk about what they were trying to achieve. The public presentation of the solution, however, was entirely performed by the teacher, with each step clearly drawn and accompanied by lengthy explanations.
**Task Six: Shanghai School 1 – Lesson 10 (the Coefficients Task)**

**Educational Context of the Task**
The class was learning about the system of linear equations in two unknowns. The students had spent previous lessons discussing the concept of solution sets, the relationship between the solution of a system and the solutions of the individual equations, the representations of linear equations on the Cartesian plane and the solution methods involving substitution and elimination.

This was the tenth lesson in this topic and one of the teacher’s goals for this lesson was to strengthen existing student knowledge of solution methods suitable for simultaneous equations.

**Social Performance of the Task**

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Mathematical Task as Stated</th>
<th>Student</th>
</tr>
</thead>
</table>
| The teacher presents the class with a task for all to consider. The teacher explains the question and invites students to suggest solution strategies. | Students A and B try to solve a system of linear equations \[ ax + by = 2 \]
\[ cx - 7y = 8 \] | (Students attentive) |
| The teacher asks class whether Bridy's is correct. | Student A finds the correct answer \[ x = 3 \], however, \[ y = -2 \] | |
| The teacher asks a third student what he thinks. | Student B copies the wrong value of \( c \), which resulted in the answer \( \begin{cases} x = -2 \\ y = 2 \end{cases} \) | |
| The teacher announces that he will proceed by combining the suggestions of Bern and Bandson. | Find the correct values of \( a \), \( b \) and \( c \). Find the incorrect value of \( c \). | |
| The demonstration of the solution to this task involves the following key steps: Substituting the correct values of \( x \) and \( y \) into the system of | | |

(Bern suggests substituting both student solutions into the system of equations.)

(Bandson suggests substituting the correct values of \( x \) and \( y \) into the equation \( cx - 7y = 8 \). This will give the correct value of \( c \).)
FUNCTIONAL ANALYSIS OF MATHEMATICAL TASKS

\[ \begin{align*}
3a - 2b &= 2 \\
3c + 14 &= 8 \\
\end{align*} \]

determine \( c = -2 \).
Substituting the incorrect values of \( x \) and \( y \) into the first equation to create a third equation
\(-2a + 2b = 2.\)
Using the first and third equations
\[ \begin{align*}
3a - 2b &= 2 \\
-2a + 2b &= 2 \\
\end{align*} \]

determine \( a = 4 \) and \( b = 5.\)
Substituting the incorrect values of \( x \) and \( y \) into the second equation to create a fourth equation
\(-2c - 14 = 8, \) thus the incorrect value of \( c \) is
\(-11.\)

Interpretive Reflection
The Coefficients Task appeared to be an intellectually demanding exploration of the students' understanding of simultaneous equations. The task certainly required more than the replication or use of a standard procedure by requiring the students to treat the coefficients of the standard format as variables and the variables as knowns (by virtue of the stated solutions).

The teacher invited students to suggest possible solution strategies; three students dutifully responded. The teacher then demonstrated the solution to the problem step-by-step, utilising elements of the methods suggested by two of the students.

The extent to which responsibility for the development of this particular solution was intended to be devolved to the students is uncertain. However, the teacher has commented in interview that "I stress on having students be the active agent in the class... the conclusion is not told by the teacher. It is through teacher's guidance, the students observe and operate... then they get the conclusion. It is like this."

The complexity of the task and the evident complexity of the teachers' decision making warrant further investigation to establish the teacher's intentions at each stage in the performance of the task.

Task Seven: Japan School 3 – Lesson 1 (the Long Task)

Educational Context
This class was learning about the system of linear equations in two unknowns. This was the first lesson in the topic and the teacher's goal for this lesson was to demonstrate how to solve a system of linear equations in two unknowns by elimination and substitution as well as to help students grasp the exact meaning of solving equations.

Social Performance of the Task

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Mathematical Task as Stated</th>
<th>Student</th>
</tr>
</thead>
<tbody>
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</tbody>
</table>

The teacher presents the class
with a system of linear equations to solve.

The teacher invites Kori to the board to present his solution.

\[
\begin{align*}
5x + 2y &= 9 \\
-5x + 3y &= 1
\end{align*}
\]

(Students attentive)

Kori writes:

\[
\begin{align*}
5x + 2y &= 9 \\
-(-5x + 3y &= 1) \\
5x + 5x + 2y - 3y &= 9 - 1 \\
10x - y &= 8 \\
10x &= 8 + y \\
x &= \frac{8 + y}{10} \\
5x + 2y &= 9 \\
5\left(\frac{8 + y}{10}\right) + 2y &= 9 \\
8 + y + 4y &= 18 \\
y + 4y &= 18 - 8 \\
5y &= 10 \\
y &= 2 \\
5x + 2y &= 9 \\
5x + 4 &= 9 \\
5x &= 5 \\
x &= 1
\end{align*}
\]

The teacher asks students what their thoughts are with respect to Kori's solution. He then invites Kori to explain his solution.

The teacher asks students what their thoughts are with respect to Kori's solution. He then invites Kori to explain his solution.

Kori's explanation involves describing each step.

The teacher once again questions students on their thoughts.

(While Suzu writes at the board the teacher conducts a discussion with the class about Kori's solution. His comments invite the students to reflect on the different solutions and how one may be an improvement on)

Suzu responds that there may be a simpler equation and is subsequently invited to the board to present his solution:

Suzu writes:

\[
\begin{align*}
5y &= 10 \\
y &= 2 \\
5x + 2 \times 2 &= 9
\end{align*}
\]
The teacher asks Endo to reproduce his solution on the board. The class is invited to “check” the solution. The teacher proceeds to highlight the unusual 0=0 result and asks, “Why did this happen?”

The bell rings so he adjourns the discussion to the following lesson.

\[
\begin{align*}
5x + 4 &= 9 \\
5x &= 9 - 4 \\
5x &= 5 \\
x &= 1 \\
\end{align*}
\]

The students are given time to replicate both solutions in their notebooks and reflect on two solution processes.

Endo writes:
\[
\begin{align*}
5x &= 9 - 2y \\
x &= \frac{9 - 2y}{5} \quad \text{(equation 3)} \\
\left(\frac{9 - 2y}{5}\right) + 2y &= 9 \\
9 - 2y + 2y &= 9 \\
-2y + 2y &= 9 - 9 \\
0 &= 0 \\
-5x &= \frac{9 - 2y}{5} + 3y = 1 \\
-\left(\frac{9 - 2y}{5}\right) + 3y &= 1 \\
-9 + 2y + 3y &= 1 + 9 \\
5y &= 10 \\
y &= 2 \\
\end{align*}
\]

\[
\begin{align*}
5x + 2x + 2 &= 9 \\
5x + 4 &= 9 \\
5x &= 5 \\
x &= 1 \\
\end{align*}
\]

Interpretive Reflection

In this task, the seemingly simple pair of simultaneous equations \[\begin{align*}
5x + 2y &= 9 \\
-5x + 3y &= 1 \\
\end{align*}\] engaged the class for a fifty-minute lesson (and indeed was the discussion point for the first fifteen minutes of the following lesson). A feature of the performance of this task was the extent to which student suggestions, responses and the articulation of their thinking were regarded as instruments for developing understanding.

The teacher, in an interview, when asked about his aim for this lesson responded, “It’s not that I just wanted them to just solve the problems but also, um, I wanted to teach them that there is a need to think about it a little – what solving equations is all about.”
### Task Eight: Australia School 1 – Lesson 8 (the Coin Task)

#### Educational Context of the Task

This is the eighth lesson in a series of fifteen lessons concerned with perimeter and area and particular emphasis is placed on the use and definitions of mathematical terms.

The teacher identified her global aims for the entire lesson sequence, as representing mathematical activity as a real-life activity and demonstrating how it relates to real-life situations.

This particular lesson was designed to focus on: i) finding the perimeter of a semi circle and quarter circle; ii) developing rules for the perimeter of non-common shapes; and iii) using student understanding and knowledge of perimeter to solve a non-routine, challenging problem.

#### Social Performance of the Task

**Teacher**

The teacher sets up the task by stating, “Now, I’ve got another question I’d like you to do... I want you to discuss it with the person sitting next to you, and see what solution you come up with.

She proceeds by drawing the following diagram:

A verbal explanation of the problem follows with students asking questions and the teacher responding. Additional information is added to the diagram, namely that the side length of the square is 25 cm (this follows that the perimeter of the square itself is 100 cm) and that the radius of the circle is 2 cm.

The explanation involves the use of a cardboard cut-out of a ‘coin’ with a hole in its centre.

#### Mathematical Task as Stated

A coin of radius 2 cm is rolled around a square of side 25 cm, always being in contact with the edge of the square. Find the distance travelled by the centre of the coin when the coin has travelled once around the square.

(diagram used in textbook from which this task was taken, but not used by the teacher)

**Student**

(Some students copying previous worked example from the board, others attentive to the new task. At one point the teacher requests that all students place their pens down and focus on the explanation of the given task.)

Students contributing comments and questions.
the teacher uses this manipulative to indicate that along the side of the square as the “coin” rolls the centre’s path is a straight line.

The teacher now writes, “Find the distance travelled by the centre of the coin when the coin has travelled once around the square.”

While the students work on the problem the teacher approaches each student in turn to see how they coped with their homework. This activity was interspersed with statements of advice to the whole class, examples include: “Have a look at how the centre of the circle moves as it goes round a corner. How would the centre of the circle move?” “The important thing to understand, or what you need to realise, is what happens when the circle goes around a corner.”

The teacher also invited students to use the manipulative at the board if they needed to.

Marcus is invited to the board and draws some of the path, along the side of the square.

Jessica approaches the board and adds to the initial drawing to indicate the path taken by the centre of the circle. Her workings result in the following diagram:
The teacher comments on Jessica’s diagram to the whole class, “Jessica’s come to the board, and from what I can tell, she’s drawn how the circle travels...you’re telling me the circle gets to the edge, and it keeps coming out, and then it drops down.”

Leon is invited to the board. His forearm is used to indicate the radius of the coin, while his fingertips indicate the path of the centre of the coin and his arm is rotated around the square’s edge with the help of the teacher.

<table>
<thead>
<tr>
<th>The teacher continues to assist students at their desks with this task. A number of students are able to arrive at a correct solution.</th>
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</thead>
<tbody>
<tr>
<td>(following lesson)</td>
</tr>
<tr>
<td>During the following lesson, after the students have attempted and corrected other problems related to perimeter the teacher gives a full solution of this task to the class.</td>
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</tbody>
</table>

Leon responds that the path taken around the edge, by the centre of the coin, is a quarter circle. He amends Jessica’s diagram to highlight this discovery:

![Diagram](#)

Adrian is invited to the board to assist in marking out the path of the coin and to discuss the exact calculation.
FUNCTIONAL ANALYSIS OF MATHEMATICAL TASKS

**Interpretive Reflection**

The Coin task was chosen for its visual appeal, for the attempt at modelling the problem with a manipulative and for its non-routine nature. The teacher presented this problem as an "additional" task – as it seemed that those students who hadn't completed their homework were to spend the time in the lesson doing 'both'. Having the students work on their homework and the coin task gave the teacher an opportunity to move around the room and to talk with each student to determine how they were coping with the work on perimeter.

Interestingly, the task was amended slightly from how it was presented in the textbook. The diagram in the textbook indicates the path of the coin’s centre, and is clearly shown to be non-linear. The teacher chose to change the task slightly by not providing a visual cue for the path taken by the coin’s centre around the corner of the square. The teacher tries to give verbal clues and tries to direct their attention to the path taken around the corner but it appears few students are able to determine this for themselves. Jessica’s diagram gives the teacher an opportunity to address their misunderstandings.

In the end, Leon’s use of his arm to model the movement undertaken by the coin successfully illustrated the curved nature of the coin centre’s path. Some students still appeared to struggle with the actual calculation itself, while others appeared unwilling to record any steps in their books and made use solely of the calculator.

There are no summarising comments made at the end of the lesson. Indeed, it is unclear what is expected of the students, if anything, before next lesson. The teacher returns to this problem in the following lesson and gives a full, detailed explanation of the solution. Although this task was chosen for its non-routine characteristics, it appeared that the majority of the students of the class were not quite ready for the degree of sophistication of the task and had not yet mastered the mathematical skills required.

**Task Nine: U.S.A. School 2 – Lesson 9 (the Compare and Contrast task – student presentations)**

**Educational Context**

This was the ninth lesson in a series of lessons concerned with functions and relations. In the previous lesson, the students were asked to complete tabulations of corresponding x and y values, in which the x-values change (-2, -1, 0, 1, 2) for the functions $0x + 3y = 6$ and $2x + 0y = 6$. The teacher discussed the slope of the graph and introduced the term "undefined slope". She also led a discussion involving the similarities and differences between the two graphs. This discussion appeared to model the type of activity she was expecting from the Compare and Contrast task.

The teacher stated in the questionnaire data that this particular lesson was designed to focus on: i) extending the students' knowledge of linear and non-linear graphs; and ii) seeing how the structured vocabulary studied in the prior lessons "crystallised." She also expressed an additional purpose for the lesson: that is, "to take a broad look at what algebra may be about (rates of change) before plugging in discrete, "small" understandings and procedures."
CARMEL MESITI AND DAVID CLARKE

The Compare and Contrast task was assigned in the previous lesson (lesson 8). In order for students’ work to contribute to answering the question, “How are the following pairs of functions alike and different?” students were asked to complete one of the following:

- a) draw up T-charts (tabulations of corresponding x and y values);
- b) draw the graph;
- c) find similarities;
- d) find differences;

for one pair of functions on a small, portable whiteboard.

(Lesson 9)

The task is written up for reference on paper on another smaller board.

She invites the students who worked on the first and second pair of functions to place their mini-whiteboards up against the main board for display.

The students who worked on the third pair of equations are invited to the front. They hold their workings in front of them and the teacher asks them to, “Tell me about yours…. Tell me what you graphed.”

The teacher comments that

<table>
<thead>
<tr>
<th>Social Performance of the Task</th>
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<tbody>
<tr>
<td>Teacher</td>
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</table>
| The Compare and Contrast task was assigned in the previous lesson (lesson 8). In order for students’ work to contribute to answering the question, “How are the following pairs of functions alike and different?” students were asked to complete one of the following:
- a) draw up T-charts (tabulations of corresponding x and y values);
- b) draw the graph;
- c) find similarities;
- d) find differences;

for one pair of functions on a small, portable whiteboard. | How are the following pairs of functions alike and different? |
<table>
<thead>
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<tbody>
<tr>
<td>i) ( y = 3x + 2 ) and ( y = -3x - 2 )</td>
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</tr>
<tr>
<td>ii) ( 0x + 3y = 6 ) and ( 2x + 0y = 6 )</td>
<td></td>
</tr>
<tr>
<td>iii) ( y = x^2 ) and ( y = \frac{1}{x} )</td>
<td></td>
</tr>
<tr>
<td>iv) ( y = 1 - 2x ) and ( y = 1 - x^2 )</td>
<td></td>
</tr>
<tr>
<td>v) ( 2y = x ) and ( y = 2x )</td>
<td></td>
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</table>

(students waiting, attentive, some propping their work up against the board and then taking their seats)

The student responds that he graphed \( y = \frac{1}{x} \) and that it turns out to be a hyperbola.
the function produces a curve but does not appear to have the form of a graph involving squaring or a higher power of x.

Another student adds that his task involved drawing the parabola, \( y = x^2 \) and adds that it touches at the origin.

A third student contributes her "similarities" – both equations are non-linear, both are a type of 'bola' and both are functions.

The teacher explains the student's use of the term 'function' by adding that she could draw a single vertical line anywhere on the graph and it would never touch the graph in more than one place.

The teacher then asks them to explain why the graph \( y = \frac{1}{x} \) is discontinuous at the origin. She then comments on the difficulty of determining the shape of a hyperbola with only a limited number of points.

Lastly, the students that worked on the fourth pair of functions are invited to discuss their findings.

The teacher makes some additional comments on the graphs regarding slope, linear decay and refers to the parabola as negative.

A student responds that both are functions and both have the same y-intercept.

Another student responds that the differences include that one is linear, the other non-linear.
The Compare and Contrast task was chosen because of the open-ended nature of some aspects of the task (that is, to list similarities and differences) and because the sharing of students' work was essential for a full discussion to take place. The task required a level of collaborative activity during the whole class discussion and, in carrying out this particular task, the teacher appeared to value written and oral contributions equally.

An interesting feature was that most of the written work had been completed in the previous lesson. The teacher had also modelled the sort of discussion she was hoping to achieve with another pair of functions in the previous lesson. Despite the well-prepared workings on the students' hand-held whiteboards, much of the discussion and contribution of knowledge during the presentations was undertaken by the teacher. Students made short responses and the teacher accompanied these with much commentary. The task was distributed in the sense that all students needed to relate their workings to another's if they were to engage with the task. As performed, the extent of the distribution of responsibility for knowledge generation to the students was limited.

DISCUSSION

Our conception of the teacher/student/task triad accords significant agency to each in the determination of the actions and outcomes that find their nexus in the social situation for which the task is the pretext. It is useful to review some aspects of the tasks described in this paper, particularly from the perspectives of the distribution of agency and voice among the classroom community.

**Task One: Japan School 1 – Lesson 1 (the Stairs Task)**

The level of student involvement in the task was high and the activity was communal and collaborative in character. The emphasis was on the development of understanding and the entire lesson time was devoted to utilising the visual representation to generate the algebraic representation. The teacher's orchestration of the several student contributions provides a useful illustration of the sort of teacher-managed distribution of responsibility that problematises the simplistic dichotomisation of classrooms into teacher-centred and student-centred (Clarke, 2006b; Mok, 2006).

**Task Two: Shanghai School 2 – Lesson 8 (the Numbers task)**

The task's focus on the establishment of the equations without the need to actually solve them added emphasis to the teacher's prioritisation of the underlying structure of algebraic representations and the development of fluency in the use of technical language. In the performance of this task, the teacher's evaluative authority was clear and, despite the several student contributions, the actual devolution of agency to the students was quite limited.
Task Three: Sweden School 1 – Lesson 11 (the Graph Task)

It is not uncommon for a task to involve constructing a mathematical model of a real-world situation. In this task, the progressive elaboration of the graph generates a context for the initial abstraction. While the teacher remained the primary authority in the construction of the context, there was some attempt at devolution of responsibility towards the students. Further analysis of the teacher interviews was required to determine whether this devolution was intended or perceived as such. In the interview, the teacher explained this approach to classroom discussions:

We did something called the understanding of what is asked in the problem. Then we solved stuff in groups. It was done mostly to get them to talk. And I am to blame for this a lot, they talk a lot but many of them talk really well [laughter] they discuss math and they figure it out in a good way together, so it is more fun for many of them especially the girls but many guys also but mostly the girls they help each other with solutions really well and I am very satisfied with their work.

Task Four: Shanghai School 3 – Lesson 7 (the Train Task)

The Train Task takes the more familiar route from real-world context to algebraic representation. To us, it appeared a novel feature of the task that the teacher chose not to emphasise the utility of the algebraic representation in solving a problem that the students could clearly answer by other means. Rather, the teacher emphasised that both the situation and the method of solution can be modelled algebraically. In relation to distributed agency and voice, it should be noted that the teacher’s approach explicitly affirms the students’ ability to generate a correct answer using non-algebraic methods and then asserts his curricular authority by interpreting the students’ methods in algebraic terms. The legitimacy of the student’s initial solution is not contested, and the teacher’s subsequent solution should be seen as an act of re-interpretation rather than appropriation.

Task Five: Japan School 2 – Lesson 1 (the Equivalent Deformation task)

Both authors, influenced by our experience of Australian mathematics classes, found the deformation task novel. We have been assured that the problem of how to change the shape of a given geometric figure, while keeping the area constant is a standard procedure in the Japanese mathematics curriculum, and it may be elsewhere, although we found no evidence of any similar tasks in the data set. The social performance of the task involved a form of Kikan-Shido (Between Desks Instruction, see O’Keefe, Xu, & Clarke, 2006) in which the teacher elicited the students’ thoughts and methods through dialogue with individual students. It was clear from the formality of the teacher’s presentation of the solution that what was being taught was intended as a standard procedure rather than a novel
investigation. The combination of distributed student and teacher performance (performed through Kikan-Shido) and teacher summative task performance is highly similar to the practices evident in many of the Australian lessons. There was little devolution of agency to the student and the emphasis of the lesson was on the correct implementation of a mathematical procedure.

**Task Six: Shanghai School 1 – Lesson 10 (the Coefficients Task)**

This task offers an interesting illustration of the use of a non-routine task to interrogate established procedures. The extent to which the responsibility for generating the complete solution might have been devolved to a greater extent to the students is a matter of speculation. The teacher's intentions and conscious decision-making could usefully be juxtaposed with the students' interpretations and consequent learning. This is a further level of analysis that we plan to carry out.

**Task Seven: Japan School 3 – Lesson 1 (the Long Task)**

Student performance 'out the front' has been discussed elsewhere (Jablonka, 2006). It is a common procedure in some countries (Japan and the Czech Republic, for example) and much less common in others. The extent to which such public performances represent a devolution of responsibility to the student, with an associated amplification of agency and voice, depends significantly on the teacher's self-positioning as the evaluative authority of what is produced by the students. In this task, significant evaluative responsibility remained in the hands of the class. The solutions being compared were, of course, entirely student-generated. It must be noted that this is the first lesson in the topic, rather than a summative lesson intended to review content already taught. The teacher's articulated goal of wanting to teach them "what solving equations is all about" was very evident in the documented task performance. The lesson is distinguished by the teacher's use of a relatively simple (and ultimately routine) mathematical task to facilitate class discussion of what it means to solve a pair of simultaneous linear equations.

**Task Eight: Australia School 1 – Lesson 8 (the Coin Task)**

The classroom performance of the coin task drew on several familiar classroom protocols. Certainly, the task was sufficiently challenging to be considered a non-routine problem for the students and there was a consequent need for student exploratory activity. The teacher made extensive use of Kikan-Shido (between desks instruction) during this exploratory activity and the manner in which this was performed was a signature characteristic of this teacher's practice. The teacher's decision not to use the illustration in the textbook placed an additional interpretive responsibility on the students that some were clearly unable to meet. However, it was the omission of this illustration that gave the task its problem-solving character. The role of visual imagery in the mathematical modelling of the task
situation was a key feature and the lesson’s dependence on a key contribution from one student (Leon) represented a meaningful devolution of responsibility from the teacher.

**Task Nine: U.S.A. School 2 – Lesson 9 (the Compare and Contrast Task – student presentations)**

This task delegated some responsibility to the students to construct distinctions between equation types and also gave significant opportunity for student voice in the articulation of the constructed differences. Given the level of preparatory modelling by the teacher during the previous lesson, the actual devolution of responsibility for knowledge generation was lower than might be thought from an examination of the task as performed in this one lesson. However, notwithstanding the level of preparation, it was clearly the teacher’s intention that the students should exercise significant agency in the construction of the differences between their assigned pairs of equations and that they should then articulate these differences in mathematically appropriate language.

While the classrooms studied both in Australia and the U.S.A. used a variety of task types, the most common form of communal task performance (involving both teacher and students) resembled either Task Five (distributed student performance followed by teacher summative performance) or an emphasis on procedural fluency. Having said this, the practices of the Australian and U.S.A. classrooms gave significant emphasis to student voice, without, however, according students significant responsibility or agency in the social development of new knowledge (cf. Sekiguchi, 2008).

It must be acknowledged that the selection of ‘distinctive tasks’ for discussion in this paper was inevitably a culturally-situated selection, carried out primarily by one of the authors (Mesiti). Nonetheless, we feel that the classroom task performances documented in this paper offer some interesting insights into the distribution of voice and agency in the classrooms of competent teachers and into the manner in which the task constrains and affords certain performances. What is clear is that classroom task performance is significantly influenced by teacher intention. This seemingly superficial observation is worth noting. The availability of ‘good tasks’ (and we would suggest that the tasks cited here have at least the potential to be good tasks, likely to promote student learning effectively) does not prescribe classroom task performance, which is a social consequence of many elements. There is no doubt that Task One (the Stairs Task), for example, would have been performed differently in the hands of a different teacher with different students.

**CONCLUSIONS**

Tasks have long been recognised as crucial mediators between mathematical content and the mathematics learner. In a very real sense, the tasks employed in the mathematics classroom represent a model of mathematics as it is performed in that
classroom. The activity that arises as a consequence of a student’s completion of a task is itself a constituent element of the learning process and the artefacts (both conceptual and physical) employed in the completion of the task serve simultaneous purposes as scaffolds for cognition, as repositories of distributed cognition, and as cognitive products. Task selection by teachers represents the initiation of an instructional process that includes task enactment (collaboratively by teacher and student) and the interpretation of the consequences of this enactment (again, by teacher and student). In this chapter, we have examined the function of nine mathematical tasks taken from classrooms in five countries. The tasks to be analysed were selected either because they appear to be typical of the mathematical activity of that classroom or because they offered a distinctive (and possibly unusual) model of mathematics, not evident in other classrooms.

We have characterised the tasks selected in each classroom with respect to intention, action and interpretation. The significance of differences between social, cultural and curricular settings, together with differences between participating classroom communities, challenges any reductionist attempts to characterize instructional tasks independent of these considerations. Rather than aiming to characterize the typical task used in any classroom, we have employed ‘distinctive tasks’ as our entry point in an investigation of what characteristics of distributed responsibility and of voice and agency are evident in the use of each particular task. Of particular interest in our analysis were differences in the function of mathematically similar tasks, dealing with similar mathematical content (those relating to systems of linear equations), when employed by different teachers, in different classrooms, for different instructional purposes, with different students. An essential tool for our analysis is a tabulation of the details related to the social performance of the task. Using these tables, our analysis drew on the video-stimulated, post-lesson interview data to identify intention and interpretation and relate both to social performance of the task.

If, as has been suggested in the last section, the classroom performance of a task is ultimately a unique synthesis of task, teacher, students and situation, then what can be learned from the analysis of such idiosyncratic social performances? As Umberto Eco is reputed to have said, “Only the ephemeral is of lasting value” and it is the transient, fleeting and not-to-be-repeated social performances that constitute the daily occurrence of the mathematics classrooms of competent teachers. We would argue that the conception that the community-at-large holds of the mathematics classroom is intrinsically bound up in the type of tasks that characterise such settings. And this conception is not in error. Mathematical tasks are the embodiment of the curricular pretext that brings each particular set of individuals together in every mathematics classroom. In other contexts, individuals come together to engage in musical performances or dramatic performances. The performances of the mathematics classroom are largely the performance of mathematical tasks and if we are to understand and facilitate the learning that is the ostensible purpose of such settings then we must understand the nature of the performances that we find there.
FUNCTIONAL ANALYSIS OF MATHEMATICAL TASKS

We contend that insight into the nature of such performances may be most evident in the comparison of the classroom performances of distinctive mathematical tasks, since the very distinctiveness (as we have defined it) of the tasks and the associated performances is likely to throw into sharp relief the social dimensions (intentions, actions, interpretations, and consequences) that characterise and distinguish effective and less effective practice. While we are convinced that post-lesson interview data demonstrate the effectiveness of the documented task performances by any conventional standards, differences in the nature of the resultant student learning outcomes arising from the different task performances could be profound. A later analysis will explore these differences.

The thread that we have pursued through the examples discussed has been that of the social distribution of responsibility, agency and voice. We commenced our analysis disposed from other studies to believe that these issues were important. Our exploration of responsibility, agency and voice in the context of the classroom performance of mathematical tasks suggests to us that competent teachers of mathematics (within the constraints of culture and curriculum) share a belief in the importance of these elements. Analysis of the classroom performance of distinctive tasks offers insights into what constitutes valued performance in that classroom. This valuing could relate to the pedagogical value accorded to particular types of classroom activity or to the mathematical value of particular activities. The valuing of agency and voice is evident in the task performances in the classrooms of these teachers, rather than in any explicit articulation by them in classroom video data or in interview.

If we are to find pattern and structure in the profound diversity of ‘well-taught’ mathematics classrooms around the world, then the attention given by competent teachers to student voice and student agency, and the mathematical tasks that they employ to catalyse that voice and agency, may represent a useful entry point for analysis.

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